Engineering Design I: Methods and Skills
Topic Readings

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Chapter 5

Geometry, Stength, and Mass Design Loop

In past readings, we reviewed techniques for **analyzing** loading, peak stress, and failure of components. In this chapter, we will begin to address the more interesting problem of **designing** good components that will not fail. Parts that don't fail meet the minimum criteria for being useful, however in order for a part to be considered "good" it should be optimized for an outcome of interest such as mass. In general, component design problems have many constraints, e.g. loading, component life or material, and can have complex objective functions including terms such as mass or cost. More complicated subjective outcomes such as aesthetics or feel may also act as constraints. More constraints typically reduce the design space and make discovery of optimal solutions easier, while more elements in the cost function typically create a more complicated cost landscape and make optimization harder.

A common design problem encountered by mechanical engineers is to select component geometry that provides sufficient strength with little component mass, given a load and material. In other words, the *free design parameters* are geometric, the *cost function* to be minimized is component mass, and *constraints* are placed on loading, material, and some aspects of geometry, such as load application points or manufacturing limitations. We start these geometric designs with a blank slate; infinite qualitative geometries could be chosen, some of which will be more suitable than others. This is quite different from the analysis problems we have discussed so far, in which you have determined the factor of safety for a part with given geometry and parameters. Analysis can be seen as a sub-task of the design process, in which candidate geometries are quickly evaluated for comparison or improvement. In the case of geometry-strength-mass design, we might compare the mass of various shapes that have each been optimized. As we become skilled, intuition will help us to guess at good candidate solutions quickly.

In later chapters, we will learn to expand to more complex design spaces by including material and loading as free design parameters.

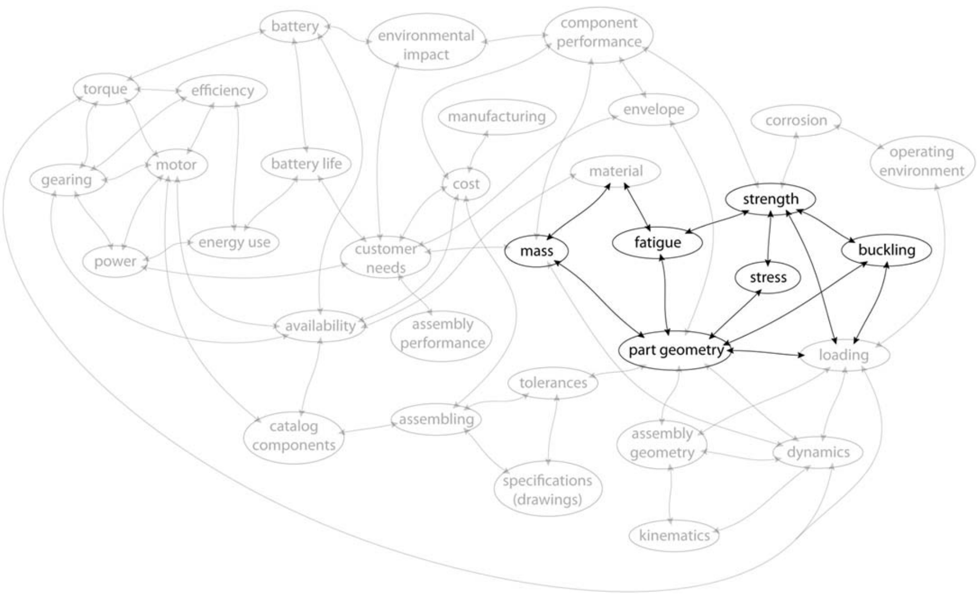


Figure 7.1 Constrained design space involved in the geometry-strength-mass design (sub-) loop. In this problem, material and external loading properties are set in advance (constrained), part geometry can be chosen freely (design parameter), and various analyses, such as for peak stress or buckling, will allow us to optimize candidate designs under the requirement that the part does not fail.

This optimization problem can also be described as follows, if you prefer a mathematical definition:

$$\begin{gathered}\underbrace{\text{minimize}}_{G}\; m = f(G) \; - \; or,\ equivalently,\ minimize\ V = f(G) \\ \text{subject to } F,\ S_y,\ E,\ FOS. \end{gathered} \tag{5.1}$$

where G is the part geometry, V and m are the volume and mass, F is loading, S_y and E are the material yield strength and Elastic Modulus (include additional properties as needed), and FOS is the factory of safety you chose *a priori*. Or, to summarize this class of design problem again:

- **Constraints**: loading, material, manufacturing, F.O.S.
- **Free parameters**: geometry (infinite possibilities with infinite variables)
- **Outcomes**: mass (or, equivalently, volume)

5.1 Designing Good Component Geometry

Now that we have defined the problem clearly, how do we actually find the best component geometry? This is a tricky problem, because there are infinite possible solutions we could theoretically consider. Take the myriad bridge designs that traverse Pittsburgh's rivers as an illustration; each is a good solution to a similar problem, but each uses qualitatively different, often quite different, geometry due to small differences in constraints or outcomes of interest.

The initial response to such open-ended, conceptual design problems describes a **creative process**. It raises the question "where do ideas come from?" Rigorous or formulaic solutions to such problems are few and impractical. Instead, each of us draws upon intuition, personal experiences, and environment for inspiration.

If you have trouble getting started, try engaging in overt brainstorming. Think about past problems you've worked on that were similar to this one, consider designs you may have seen in related machines, or research common designs for things with similar characteristics. You might even try picking up a mechanical object or visiting a machine shop; you never know what stimuli might lead to a good design idea. In the case of geometry-strength-mass design problems, you might try out shapes that involve triangles and circles, which often seem to do well, and might avoid bending moments and sharp (internal) corners, which often produce large stresses. This process is not guaranteed to produce the best candidate designs, since no process will do so, but it will give you something to start with and begin learning from and refining.

The key to good design is **iteration**, especially when your intuition is still forming. Complete solutions do not arise immediately after hearing the design problem. Instead, during the conceptual design phase, expect to try out many candidate designs, identify weak aspects, and either refine them or try something totally different. In the case of geometry-strength-mass design problems, "try out" would mean: perform a Free-Body Diagram analysis, stress analysis, and other pertinent failure analysis (e.g. buckling) as necessary; and solve for the design parameters that minimize mass in that qualitative design. Keep in mind that large forces often lead to large stresses, so a free body diagram by itself can tell you a lot about the strength of a shape. As you perform such analyses on different designs in succession, trends will emerge that will allow you to converge on favorable geometric features, i.e. shapes that support the load with little mass.

After you have formed a strong theoretical foundation for your overall design through iterative simple-model considerations of candidates, iterations start to include computational modeling. If initial CAD results differ greatly from your expectations, go back to simpler analytical models, identify weak assumptions, and re-iterate. In geometry-strength-mass design problems, this might include improving assumptions for (negligible) reaction loads or the dominant mode of failure. When CAD results begin to look as you expect, begin to iteratively refine and tweak design details in your CAD model. In geometry-strength-mass design problems, this might mean eliminating volumes of material that are not heavily stressed. Rather than adding new features, such as weight-reduction holes (which cause stress concentrations), try refining the existing feature parameters (to make things smaller) or adding simple continuous cuts (such as pockets to produce I-beam cross-sections), which often reduce mass more while producing fewer stress concentrations. Refinement might also include reducing stress concentrations, for instance by rounding out sharp internal corners. Remember that in an optimal part, all material is equally stressed. Perhaps counter-intuitively, a part that is mostly "blue" after FEA is not a very well-designed part, and may not even be a very strong part. It is best if the distribution of stress is even such that every portion of the part has the same factor of safety resulting in a very "red" part (where red corresponds to a maximum stress at the limit defined by factor of safety).

New computer-aided design tools are emerging, including tools that attempt to automate the geometry-strength-mass design process, but these are not yet powerful enough for our purposes. The current version of SolidWorks includes a part optimizer that finds values for selected component dimensions that minimize mass while maintaining a set factor of safety. However, this process is not much faster than iterative refinement by a human operator, and cannot find qualitatively different designs (different shapes). Laboratory studies using brute force optimization techniques, such as genetic algorithms, have demonstrated the capacity to optimize overall part geometry, but such tools are not yet easily accessible. Larger design problems, allowing for multiple parts or catalog components for example, are still well beyond the scope of brute-force optimization. For now, mechanical design relies on human intuition, brainstorming, iterative application of analytical tools, and iterative refinement of computer models.

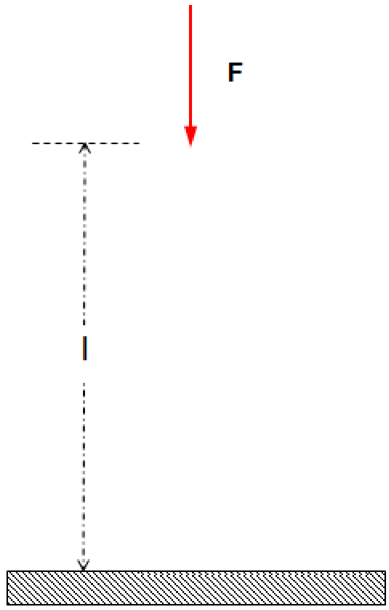
5.2 Intuitive Design Exercises

One way to improve your design intuition is to practice on simple, canonical design scenarios. Simple scenarios allow for fast analysis, and therefore more iterations in a short period of time. More complex components can often be reduced down to a set of very simple scenarios, so a strong intuition in this domain also allows faster navigation of larger design spaces. Here are a couple of simple design exercises in the geometry-strength-mass design regime.

Exercise 1: A compressive, normal load

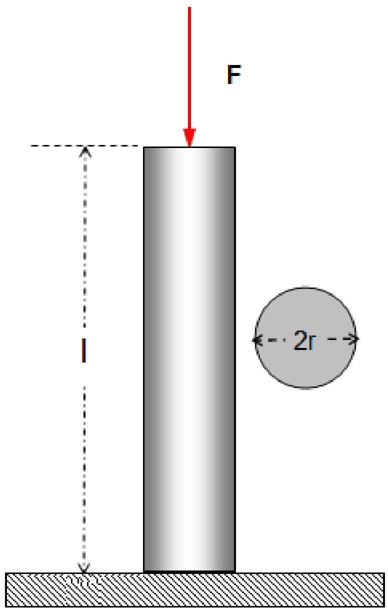
Problem statement:

A large force F is to be applied normally to a surface through a support component at a distance of l from the surface, as shown at right. Design the support component of known material (density ρ, yield strength S_y, and modulus of elasticity E) such that the overall mass of the system is minimized.



Example intuitive design steps:

1. Let's try a cylindrical column. The force is applied normal to the support component, thus, the main stress in the part will be compression. Assuming loading F is much bigger than support component weight, mg, the compression due to m is negligible compared to that caused by external loading, and the stress is approximately the same along the length of the part. The part should therefore be designed to have the same intersection area A at different horizontal levels. Since $m = \rho \cdot l \cdot A$, the area A should be minimized to minimize m. Let's ignore contact stress at the point of load application for now. The design has one free parameter, r, the cylinder radius, so it's nice and simple.



2. For this cylinder, two failure modes seem important: yield due to compression and buckling. Let's investigate these one by one.

3. The peak stress due to compression should be related to allowable stress:

$$\sigma = \frac{S_y}{FOS} = \frac{F}{A} = \frac{F}{\pi r^2},$$

Thus, the radius that minimizes mass while not yielding is:

$$r = \sqrt{\frac{F \cdot FOS}{\pi \cdot S_y}}$$

4. While we are trying to shrink r, buckling may become a problem. To avoid buckling, load F should be related to the critical force (equations for which vary with type of load application and can be found online, or in Shigley's Mechanical Engineering):

$$F \cdot FOS = F_{cr} = \frac{C\pi^2 EI}{l^2} = \frac{\pi^3 E r^4}{16 l^2}.$$

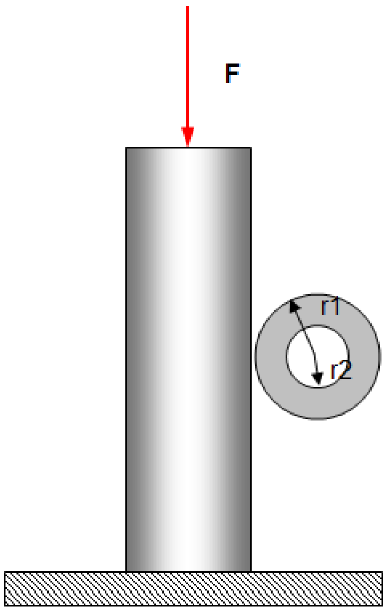
Where the boundary condition constant $C = \frac{1}{4}$

To avoid buckling, we need to ensure

$$I \geq \frac{4 \cdot F \cdot FOS \cdot l^2}{\pi^2 EI}, \text{ or } r \geq \sqrt[4]{\frac{16 \cdot F \cdot FOS \cdot l^2}{\pi^3 E}}$$

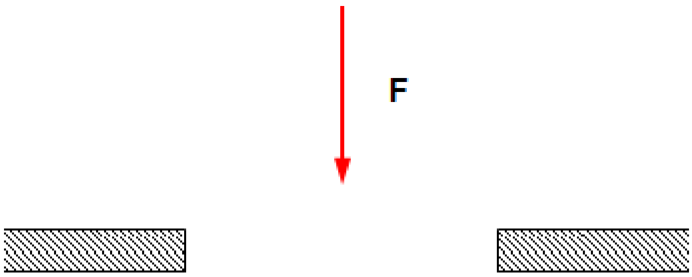
5. Depending on the values of l and F, buckling might be the dominant cause of failure, leading to a larger value of r than necessary to prevent yield in compression. Unfortunately, we only have one free parameter in our cylinder, and cannot independently vary I. Perhaps a tube, in which the cross-sectional area is concentrated far from the centroid, would satisfy both failure criteria with the same factor of safety, leading to a more effective part...

   And so on...



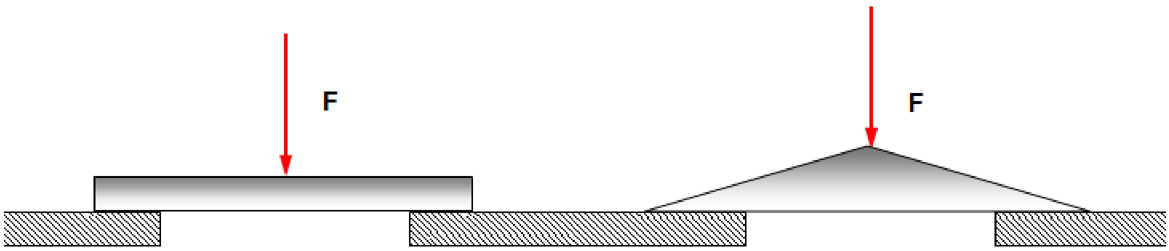
Exercise 2: Bridging a gap

Now consider the case where a load is applied over a gap in the support surface, as illustrated below. What kinds of shapes might support this type of load with minimum material?



Let's try a rectangular cross-section beam first. A quick free body diagram or two will reveal that the beam experiences a bending moment at its center. This will likely dominate stresses in the component, resulting in some minimum combination of base and height parameters (see Topic Reading 2A for this analysis). Minimum mass would be achieved with infinite height and zero base by this model, but would introduce other failure modes, e.g. buckling or tipping over, and might also lead to issues in manufacturing. So perhaps there is a practical minimum base width, leading to a perfectly constrained system with one free parameter, h, and one value that minimizes mass while meeting factor of safety requirements.

If the rectangular beam is designed to withstand this peak moment at its center, it will necessarily be over-built on its ends where the internal moment is low. Perhaps we can vary the height from the center to the ends, matching the varying internal moment. This would be a simple way to remove material on the ends that would otherwise experience little stress. And so on...



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