Engineering Design I: Methods and Skills
Topic Readings

Steven H. Collins

March 16, 2015

Chapter 8

Catalog Component Selection

Some assembly functions are best accomplished using components purchased from a catalog. Such components typically perform one specific function common to many machines, a function that usually requires relatively high complexity and precision for adequate performance. Custom designs of such elements would be expensive and time consuming, and catalog components provide efficient solutions through the economies of mass-manufacture, with the tradeoff that only a discrete set of predetermined designs are available.

Ball bearings, wire rope and machine screws are canonical examples of catalog parts. Each provides a function common to many machines; ball bearings allow low-friction rotational motion, wire ropes provide flexible tension transmission, and screws form non-permanent rigid joints. Each solution is complex; ball bearings comprise dozens of precise moving parts, wire ropes are composed of hundreds of interwoven fibers, and screws have complex thread and cap geometry. Each is easier and cheaper to purchase than to design on one's own, and in each case the set of available parts is limited by the offerings of manufacturers.

Of course, many assembly functions are not well addressed by catalog components, particularly those functions that are unique to a specific application. In fact, most components that would be desirable for a particular design problem are simply not available. In the design of a support structure for an astronaut's flight suit, for example, one probably would not be able to find just the right component to transfer loads from the wall to the helmet. Purchasing, e.g., a bracket used for shelving and retrofitting it to the helmet would likely result in a bulky, heavy, complex design. Designing your own component perfect for the geometry and loads of the helmet results in a much more elegant solution. The overall design would still make use of catalog components such as screws, standoffs or washers to connect the custom component to the wall.

Another way of thinking about the relationship between catalog components and custom components is that catalog parts perform the most difficult, but common, tasks, while custom parts hold everything together nicely for one application.

Component types that are usually available via catalog are not always available with the desired properties. In some sense, catalog components are always suboptimal due to physical limitations. For example, we would prefer a ball bearing with zero mass, zero friction, and infinite maximum load, which is of course impossible due to the need for finite-size rolling elements and contacts. We must therefore always take such constraints into account while designing an assembly, and we will discuss strategies for common component types below. Sometimes components are not available with required properties simply because few other customers would purchase such an item. In cases where many (thousands) of components are required for a product, a manufacturer (who supplies catalogs) may be able to design and produce the component to unique specifications. In the rare case that a feasible component cannot be purchased and the design cannot be altered, one must then produce their own custom component.

Catalog component selection, as with material selection, is a case of design by selection across a discrete set. We must therefore first have some knowledge of the types of things that might be available, the types of properties that will be important for each thing, and the places we might look for options. We can then use analysis of simplified models of the system to determine acceptable values for each property or desirable sets of properties to minimize or maximize.

In this chapter, we will provide guidelines for the selection and integration of a few catalog components common to robotics: fasteners, bearings, shafts, spur gears, timing belts and pulleys, wire rope, and springs. In each case, we will discuss some simple models and heuristics that can help quickly narrow the search region, as well as suggestions for best use of such components in robot design. This is not intended as a replacement for a detailed analysis of machine components such as found in, e.g., Budynas and Nisbett [2006] chapters 7-14. There is a rich literature on "machine design", or more accurately machine *analysis*, which is very useful in the detailed design of such components by manufacturers or when designing at the limits of component performance. Instead, this document is intended to serve as a practical tool for those getting started with design in this domain, with an expectation that readers will later consult the literature for more detailed models and empirical data if necessary.

8.1 Fasteners

Fasteners are some of the most common components in mechanical engineering design, used to create rigid, non-permanent joints between parts. Here we will consider two common types of fastener: socket cap screws and nuts and bolts. In our discussion below, we will assume that you have already determined the geometry of the parts to be connected, including considerations of joint loading, error propagation, and constraint.

8.1.1 Machine Screws

Machine screws are useful general-purpose fasteners in robotics applications. The following terminology is useful in their selection and integration:

- **Threads** are the helical grooves on the outside of the screw (external threads) that allow it to interlock with the matching grooves inside a nut or threaded hole (internal threads). Most screws are *single threaded*, having one continuous groove rather than several parallel grooves, and *right-handed*, meaning that clockwise rotation will cause the screw to move into the hole.

- **Pitch** indicates the distance between adjacent threads, commonly listed as the inverse of this value, e.g. threads per inch.

- **Major Diameter** is the diameter of the outer edge of the threads, equivalent to the diameter of a hole through which the screw would just barely fit.

- **Tap (or Die) Diameter** is the diameter of the hole (or positive cylindrical feature) that should be manufactured to allow internal threads (or external threads) to later be cut into the material using a tap (or die).

An 8-32 stainless steel socket cap screw
with a hex drive and partial threads.

Threads typically conform to standards set by either the Unified Thread Standard, or UTS, for inch screws, or the International Organization for Standardization, or ISO, for metric screws. These standards for thread types, including specifications of diameter, pitch, depth, and thread angle as well as their tolerances, allow for selection of screws as a commodity, without detailed knowledge of each manufacturer's practices. Two common inch thread designations are UNC, or unified national coarse, and UNF, or unified national fine. These refer to two standard thread pitches for one standard major diameter, one with fewer, stronger threads per inch (UNC) and the other with more threads and precision (UNF).

Threads are usually specified as *D-P*, where *D* relates to major diameter and *P* relates to thread pitch. Common examples of inch thread designations are 4-40, 6-32, 8-32, 10-24, and ¼-20. Here, 4, 6, 8, 10 and ¼ are the number (gage) or inch sizes of the major diameters of the screws (0.112, 0.138, 0.164, 0.190 and 0.250 inches, respectively), while 40, 32, 32, 24 and 20 refer to the number of threads per inch. These threads are all UNC, with UNF counterparts of 4-48, 6-40, 8-36, 10-32 and ¼-28. Thread types and corresponding major diameters, clearance hole diameters, and tap diameters can be found in a *Drill and Tap Chart*.

Machine Screw Size		Threads Per Inch	Minor Dia	Tap Drills				Clearance Hole Drills			
				Alum, Brass, & Plasitcs 75% Thread		Stainless Steel, Steels & Iron 50% Thread		All Materials			
								Close Fit		Free Fit	
# or Dia	Major Dia			Drill Size	Decimal Equiv.	Drill Size	Decimal Equiv.	Drill Size	Decimal Equiv.	Drill Size	Decimal Equiv.
0	.0600	80	.0447	3/64	.0469	55	.0520	52	.0635	50	.0700
1	.0730	64	.0538	53	.0595	1/16	.0625	48	.0760	46	.0810
		72	.0560	53	.0595	52	.0635				
2	.0860	56	.0641	50	.0700	49	.0730	43	.0890	41	.0960
		64	.0668	50	.0700	48	.0760				
3	.0990	48	.0734	47	.0785	44	.0860	37	.1040	35	.1100
		56	.0771	45	.0820	43	.0890				
4	.1120	40	.0813	43	.0890	41	.0960	32	.1160	30	.1285
		48	.0864	42	.0935	40	.0980				
5	.125	40	.0943	38	.1015	7/64	.1094	30	.1285	29	.1360
		44	.0971	37	.1040	35	.1100				
6	.138	32	.0997	36	.1065	32	.1160	27	.1440	25	.1495
		40	.1073	33	.1130	31	.1200				
8	.1640	32	.1257	29	.1360	27	.1440	18	.1695	16	.1770
		36	.1299	29	.1360	26	.1470				
10	.1900	24	.1389	25	.1495	20	.1610	9	.1960	7	.2010
		32	.1517	21	.1590	18	.1695				
12	.2160	24	.1649	16	.1770	12	.1890	2	.2210	1	.2280
		28	.1722	14	.1820	10	.1935				
		32	.1777	13	.1850	9	.1960				
1/4	.2500	20	.1887	7	.2010	7/32	.2188	F	.2570	H	.2660
		28	.2062	3	.2130	1	.2280				
		32	.2117	7/32	.2188	1	.2280				

Example of (part of) a tap and drill chart.

As a screw is tightened, it becomes loaded in tension. Torque applied to the head of the screw tends to cause rotation of the threads, which tend to pull the threaded region deeper into the hole, lengthening the screw and loading it axially. To relate applied torque to axial tension, we can model the threads of the screw like a wedge (see the figure below). Torque tends to drive this wedge under the accepting part, pulling it towards the screw cap. A free body diagram reveals that the driving force on the wedge is approximated as the torque divided by the thread radius, resisted by normal and frictional force components at the thread interface. The axial components of these normal and frictional forces must then be counteracted by an axial force between the screw and accepting part:

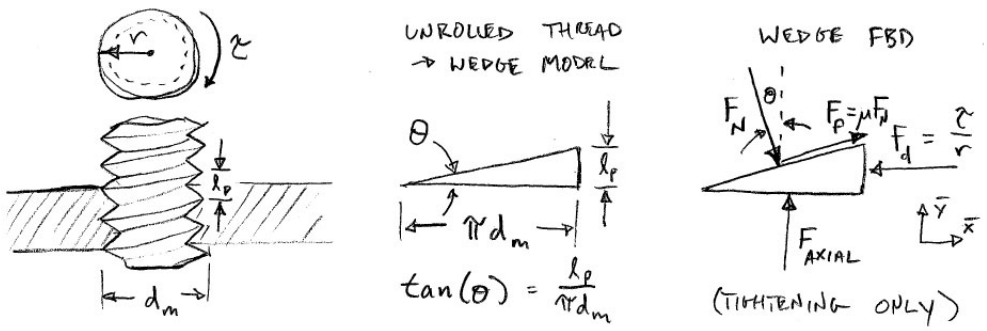
$$F_{axial} \approx \frac{\tau}{1/2 \cdot d_m} \cdot \frac{\cos\theta - \mu \sin\theta}{\mu \cos\theta + \sin\theta} \tag{8.1}$$

where τ is the torque applied to the screw, d_m is the major diameter, μ is the coefficient of friction, $\theta = \arctan(l_p/\pi \cdot d_m)$ is the angle of the wedge, and l_p is the pitch length. For typical values of μ and θ, say 0.3 and 0.07 radians, respectively, the second term evaluates to about 2.5, or $F_{axial} \approx 5 \cdot \tau \cdot d_m^{-1}$.

Note that this is only true when tightening the screw, since the friction force will act to oppose motion of the screw. If the screw is at rest, friction will act to counter the x component of the normal force caused by residual axial forces from the previous tightening, preventing the screw from spinning out. During loosening, the friction force will again resist the applied torque, while the normal force will act to help loosen the screw.

In the special case that friction is negligible, such as with or ball screw or well-lubricated power screw in a geared transmission, this relationship reduces to:

$$F_{axial} \approx \frac{\tau}{1/2 \cdot d_m} \cdot \frac{1}{\tan\theta} = \frac{2 \cdot \tau}{d_m} \cdot \frac{\pi \cdot d_m}{l_p} = 2\pi \cdot \frac{\tau}{l_p} \tag{8.2}$$



Screw torque and axial loading related by modeling threads as a wedge.

Another estimate of axial tension in a tightened screw is obtained by relating rotation of the screw to deformation of the screw and fastened part. With each rotation, the nominal length of the free portion of the screw decreases by the pitch length, l_p. The sum of the compression of the fastened part and the elongation of the screw must therefore equal $N \cdot l_p$ after N tightening rotations. Axial load can then be derived from simple models of the screw and part, typically having parameters (analogous to) elastic modulus, E, cross-sectional area, A, and resting length, L.

Axial loads induced by tightening a screw are usually helpful. When the fastened part is not externally loaded, residual screw forces will cause a reaction force between the fastened and threaded parts. This has the benefit that friction at the contact will prevent the fastened part from becoming misaligned. As the fastened part is loaded, the screw will typically not experience additional loading at first; instead, the reaction force between the fastened part and the threaded part will decrease in proportion to the applied load. Therefore, it is usually advantageous to tighten a screw until the induced axial load is moderately higher than the expected axial load transferred through the assembly.

Peak rated load is an estimate of the axial tension at which a screw will break. Failure load ratings are listed by suppliers or in UTS specifications, and depend upon factors such as screw material and heat treatment in addition to size. Failure load can also be estimated based on material properties and cross-sectional area, but caution is warranted due to the complex geometry and presence of stress concentrators. To provide a sense of the strength of common screws, the rated failure loads of Grade 2, steel 4-40, 6-32, 8-32, 10-24 and 1/4-20 socket cap screws are about 200, 300, 450, 600 and 1000 pounds force, respectively.

Another failure mode for screws in tension is failure of the thread itself, also known as stripping of the threads or pullout. A simple heuristic to avoid stripping threads is to always have at least three full threads in contact; most load transfer occurs in these first three threads regardless of the total thread number. In most situations, with three or more full threads the screw itself will fail in tension before the threads are stripped. Additional threads can be added if the material of the threaded part is significantly weaker than that of the screw. If it is important for a screw to remain perpendicular to the surface into which it is inserted, a good heuristic is to provide threads to a depth of at least two screw diameters.

Screws can also fail in shear. However, design with the intent of loading screws in shear is discouraged (for reasons discussed in the previous chapter). If a design necessitates loading in shear, estimates and ratings similar to those described for tensile loading above can be used.

Machine screws are available with a variety of head or cap types. Socket cap screws have a hexagonal (hex) cutout in a cylindrical head which accepts an allen wrench, and are a good first choice. Button head, low head, or round head screws have shorter axial head length for better clearance, while flat head screws fit into a chamfered hole for zero interference. Drives in these heads are more easily stripped than with a cap screw. Screws can be purchased with slotted, phillips, hex, or torx drivers. Hex head screws have an external hexagonal shape for tightening at higher torque with a wrench. Set screws have no head, but a hex drive in the center of the threaded cylinder itself, and are designed to make contact between the tip of the screw and an element on the other side of a threaded hole.

8.1.2 Nuts and Bolts

In most robotics applications, it is preferable to use a machine screw with a through hole and a threaded hole in a custom part rather than a bolt with a nut and two through holes. Threaded holes result in fewer parts and lower mass. Threaded holes also result in lower expected error, since they maintain screw position more precisely than a bolt in a clearance hole. In some cases, however, threading holes may be prohibitively expensive, and nuts and bolts are then a useful alternative.

Nuts and bolts are designed to be tightened strongly and precisely during installation. In a typical application, the nut and bolt drive two assembled elements together with enough force that friction between the two transfers all expected loads. During the tightening process, the nut will often yield slightly, and it is therefore best not to reuse nuts once fully tightened. Bolts nearly universally come with hex heads, and nuts typically have an external hex shape with about three full internal threads (illustrating the previously-described pullout heuristic). Concepts related to threads, standards, tensioning, and strength are otherwise very similar for bolts and machine screws as covered in the previous subsection.

8.1.3 Fastener Selection Process

The most important parameters in screw or bolt selection are material and major diameter, which together determine axial strength. To determine the required axial strength, perform a static analysis using free-body diagrams. Next consult a screw strength chart to determine an appropriate major diameter and material. Finally, consult a catalog to find a part with the desired major diameter and material, a common thread pitch, and other properties appropriate to your application such as length, head and drive type, and finish. Check that the rated strength of the final part meets your requirements, including factor of safety, and iterate as necessary.

8.2 Bearings

Bearings connect parts together while allowing relative motion. In any assembly with moving parts, bearings are used in one way or another. Bearings, or sets of bearings, can allow single or multiple degrees freedom in rotation or translation. Here we will discuss single degree of freedom rotational bearings, single degree of freedom linear bearings, and finally specialty and combination bearings.

8.2.1 Rotational Bearings

Single degree of freedom rotational bearings, used in forming 'revolute' (or 'pin' or 'hinge') joints, are the most common and well-developed type of bearing. They tend to result in simple, lightweight and otherwise high performance designs. It is therefore always advisable to consider ways of generating desired assembly motions using revolute joints. The most important properties to consider when selecting rotational bearings are strength, size, resistance, speed, and precision, which are typically reported using the following terms:

- **Radial load** is the maximum allowable force applied to the bearing in the direction orthogonal to the axis of rotation of the bearing.
- **Axial load** is the maximum allowable force applied to the (inner race of the) bearing in the direction along the axis of rotation of the bearing. It is much lower than the allowable radial load for most bearings.
- **Inner diameter**, or shaft diameter, is the diameter of the hole in the bearing (or its inner race) into which a shaft is inserted.
- **Outer diameter** is the diameter of the outer surface of the bearing, closely related to the size of a hole into which the bearing would just barely slip.
- **Width**, or axial length, is the length of the bearing along the axis of rotation.
- **Resistance** to rotation in a bearing can be modeled as Coulomb friction, $\tau_f \approx -sign(\dot{\theta}) \cdot 1/2 \cdot D_i \cdot \mu \cdot F_{radial}$, with additional viscous terms at high speeds, $\tau_v \approx -sign(\dot{\theta}) \cdot C_D \cdot \dot{\theta}^2$.
- **Rated speed** is the maximum rotational speed allowable due to the dynamics of internal rolling elements in ball bearings and roller bearings. Journal bearings have an additional lower limit on speed related to fluid dynamics.
- **Precision** of a bearing relates to the expected error in shaft position due to eccentricity, gaps or compliance. Ball bearing tolerances are standardized to the ABEC scale (Annular Bearing Engineering Committee), with higher ABEC ratings indicating better precision.

Left to right: Plain bearing (or bushing), ball bearings, and needle roller bearing.

The most common forms of single degree of freedom rotational bearings are plain bearings, ball bearings, and needle roller bearings:

- **Plain bearings**, or bushings, are often no more than a thin tube made from a material with low friction and good abrasion resistance. Plain bearings are small, lightweight and cheap and typically tolerate high loads. They are less precise than ball bearings, however, due to the need for clearance between the hole and shaft. They also tend to have higher resistance, since relative motion is achieved through sliding rather than rolling.

  Maximum radial load is usually given as a peak pressure in plain bearings, which can be related to peak radial force using the approximation $\sigma \approx F \cdot (w \cdot D_i)^{-1}$, where w is width and D_i is inner diameter. Coulomb friction provides a good approximation of plain bearing resistance, with values of μ ranging from about 0.05 for Teflon to 0.20 for lubricated brass.

  Plain bearings should be of a dissimilar material to the shaft to prevent adhesion. Common bushing materials include metals, like brass, and plastics, like Teflon. Brass is cheap and durable, but has high friction and density. Teflon has low friction, but also low strength and abrasion resistance, and is therefore often impregnated into brass or another strong backing element.

- **Ball bearings** comprise a set of rolling spherical elements housed between an inner and outer 'race'. They have very low friction ($\mu \approx 0.001$), high precision and tolerate high speeds. However, contact stresses on the rolling elements strongly limit radial load and tend to result in large, heavy designs.

  Ball bearings will quickly break if axial loads are applied to the inner race. The balls roll in a shallow groove which affords little positive traction against axial motion and acts as a force multiplier on radial load. Similarly, even small off-axis moments on the inner race will damage a ball bearing.

Shielding creates a barrier between the environment and the moving elements inside a ball bearing. This prevents damage from particles that could enter the bearing and lodge between the ball and the race, but also increases resistance, width and mass.

- **Needle roller bearings** are similar to ball bearings, with cylindrical rather than spherical rolling elements. Increased contact area between the rollers and the race (or shaft) allows needle roller bearings to accept much higher radial loads in a smaller, lighter component. These cylinders are harder to assemble with an inner race, however, and small diameter roller bearings typically do not have one. The mating shaft should therefore be prepared for direct contact with the cylinders, preferably comprising hardened, polished steel. Precision is thereby poorer in small diameter roller bearings due to the need for clearance between the rollers and the shaft during assembly.

It should now be clear that the ideal radial bearing will depend strongly on the relative importance of resistance, size, mass, precision and cost in a given application. In cases where transmission efficiency and precision are paramount, such as a gearbox, ball bearings are often a good choice. If size and mass are more important, plain and roller bearings are a better choice. If cost is critical, bushings win out. A quantitative assessment of the key properties of each type of bearing is often warranted, especially for those new to the domain. In particular, it is worth calculating the resistance torque due to friction, which is often lower than expected due to the shaft radius term, $1/2 \cdot D_i$. A common mistake is to overvalue low resistance and use a ball bearing in an application where the shaft diameter is small, the number of cycles is low, efficiency and precision are not critical, but mass and size are, and therefore a bushing would have higher performance.

The outer diameter of a bearing is usually manufactured to tight tolerances to allow seating within a custom part. Bearings can be secured by press fitting, in which case the hole should have an appropriate interference fit, i.e. a diameter a few thousandths of an inch smaller than the outer diameter of the bearing. A retaining compound can instead be used, in which case the hole should have a slip fit, which is easier to manufacture and assemble, but care should be taken not to get glue inside the bearing. Some ball bearings are designed for a particular mode of seating, which should be noted during design. To obtain suitable accuracy of holes meant to seat bearings, a reaming operation is usually required.

Flanges are a common option in radial bearings, and can help seat the bearing with precise axial position. In plain bearings made of a uniform material, the flange can have the added benefit of acting as a built-in thrust washer (see below).

Because radial bearings are either poorly suited or completely unable to react out off-axis moments, they are often used in pairs spaced along a shaft. As discussed in the previous chapter, simple support, with one bearing on either side of the load applied to the shaft, usually results in the lowest radial bearing loads.

8.2.2 Thrust Bearings

Thrust bearings, or thrust washers, are often used in combination with radial bearings to fully constrain the elements of a revolute joint. Thrust bearings take axial loads that would damage, or simply not be resisted by, radial bearings. They are usually placed such that one face contacts a custom part housing the bearing, while the other face contacts an element attached to the shaft, such as a gear or a second custom part. Thrust bearings are available with designs analogous to their rotational counterparts, including plain, ball and roller thrust bearings, with corresponding qualities. Rolling thrust bearings require an appropriate rolling surface, which can be built into a custom part or provided with separate washer elements purchased with the bearing. Whenever significant axial loads are expected, some explicit form of thrust bearing is advisable to manage friction and wear.

Left to right: Thrust washer, roller thrust bearings, and needle thrust bearing.

8.2.3 Linear Bearings

Linear bearings and rails are used to create translational joints between components. The primary sliding and rolling elements are analogous to their rotational counterparts described above, including plain, ball and roller bearings. Joints designed around linear bearings have numerous disadvantages compared to their rotational counterparts, however, and tend to result in much larger, heavier, more complex, and more expensive designs. Drawbacks include:

- **Constraining** a joint to have only one linear degree of freedom is difficult. Bearings on round rails will rotate in addition to translating, necessitating multiple rails. Rails with features that resist rotation are complex, expensive, and, by necessity, have large cross-sections. If the axial length of the linear bearing is small compared to its diameter, *binding* becomes a problem; small off-axis moments tend to drive the top rear and bottom front edges of the bearing into the rail, generating friction and grinding it to a halt. To prevent binding, linear bearings must therefore be relatively long. Reacting out larger off-axis moments, arising from offset driving and resisting elements for example, requires multiple linear bearings spaced out along each rail.

- **Recirculating rolling elements** are required in linear ball or roller bearings, since each rolling element travels at half the speed of the bearing as it translates along the rail. As a ball approaches the rear of the bearing, it is pushed out into a separate channel where it moves towards the front of the bearing with a motion similar to that of a tank tread. This makes a linear ball bearing much larger and more complex than its rotational counterpart.

Linear bearings and rails. *Top left:* linear plain bearings. *Top right:* round linear bearing with recirculating balls. *Bottom left:* round linear bearings on rails. *Bottom right:* rectangular linear bearing on a multi-featured rotation-resisting rail.

- **Large, precise rails** also add to the size, mass, complexity and cost of linear joints. The entire rail surface must be hard and smooth, since rolling or sliding elements contact the rails directly. Rails must also remain very straight to avoid binding. They often cannot be supported for most of their length, however, since supports would interfere with bearing motion. Making matters worse, side-load on the rails often places them in bending. The rails must therefore have large cross-sections to avoid deflection. Rails must be long enough to allow adequate *travel* of the joint, in addition to the nominal length of the bearing structure, and the majority of this length is unused most of the time. Compare this to a rotating shaft, where small inner races replace large polished surfaces, torsional deflection is usually not critical, and components can be stacked compactly along the shaft axis.

To summarize, a well-designed linear joint will typically require four large, complex linear bearings with a large square footprint, riding on two long, thick, polished rails, making the joint large, heavy, complex and expensive.

8.2.4 Specialty and Combination Bearings

In some applications, other types of bearings could provide an advantage. Here are a few thoughts on less-common bearings and when they could be useful:

- **Angular contact bearings** are a hybrid of radial and thrust bearings, in which the moving elements roll on a conical surface. They are designed for situations in which a consistent combination of radial and axial load are expected, such as might arise in a set of bevel gears or a lead screw driven by a spur gear. Under just the right circumstances, they can outperform the combination of a radial bearing and a thrust bearing, but usually with increased cost and lead time.

- **U-joints**, or gimbals, allow two rotational degrees of freedom. They are comprised of two revolute joints with orthogonal, intersecting axes of rotation, typically achieved via an X-shaped 'floating' inner element in simple support by two bearing sets, one on each component being joined. In most robotics applications, it is best to design a custom two degree of freedom joint, if needed, rather than incorporating a catalog gimbal component.

- **Ball joints**, or ball and socket joints, allow three rotational degrees of freedom. They are comprised of a spherical element partially captured by a spherical enclosure. Ball joints are usually used in concert with additional constraining elements, such as in a three-dimensional n-bar linkage, since they are otherwise too difficult to actuate.

- **Gantries** allow two translational degrees of freedom. They are usually created by mounting the ends of the rails of one linear joint onto a linear joint with an axis orthogonal to the first. In addition to all the ills of a single degree of freedom linear joint, a gantry system has the problem that the large, heavy rails of the first joint must move (and accelerate) as the end effector moves in the second direction.

8.2.5 Selecting Bearings

First, use the above concepts to make an educated guess as to which qualitative types of bearing are suitable for the application in terms of degrees of freedom and constraint and the relative importance of size, resistance, precision and cost. Next, use analysis of simplified models to determine the maximum radial and axial loads. Consult a catalog for candidate parts, using bearing type and radial load to narrow the search. Select among viable options based on lower-importance features such as flanges or shielding. Finally, note outcomes such as size, mass, precision and cost, and estimate maximum resisting torque based on expected loads and bearing radius (and material in bushings). Iterate through the entire process until reaching a point of diminishing returns.

8.3 Shafts

Shafts are cylindrical components used in combination with bearings to allow rotational motion, and often used in combination with elements like gears to transmit torques. Since the desired shaft length and features are unique for every application, *shafting*, or shaft stock material, is usually purchased, cut and machined, rather than purchasing fully featured components. The most important properties in shaft selection are material and outer diameter.

Shafts are often loaded in torsion and/or bending, placing a lower limit on a combination of material strength and shaft diameter. Rearranging terms from a simplified analysis of peak stress, we have:

$$S_{ys} \cdot D^3 \approx \frac{16}{\pi} \cdot T \cdot fos, \quad and \quad S_{yt} \cdot D^3 \approx \frac{32}{\pi} \cdot M \cdot fos \tag{8.3}$$

where D is the shaft diameter, S_{ys} is shear yield strength, often approximately ½ the tensile yield strength, S_{yt}, T and M are the peak torque and moment applied to the shaft, respectively, and fos is factor of safety. Shaft diameter therefore has a dominant effect on strength.

In applications where rolling elements make direct contact with the shaft, such as with small-diameter needle roller bearings or linear rolling bearings, it may be necessary for the shaft surface to be hard and precise. Shafting for these applications is typically made from steel, hardened or case hardened, and then ground and polished. This process yields an exceptional finish, but is expensive and makes the resulting shafting hard to work with. In most applications, shafting with lower surface quality performs equally well and is cheaper and easier to manufacture.

Left to right: A shaft collar with a split-hub clamp,
a gear with a set screw, and a retaining ring.

8.3.1 Rigid Shaft Connections

Shafts are rigidly connected to other components in an assembly to provide support, maintain (axial) position, or transmit torque. Here are some common types:

- **Split-hub clamps** use force from a tightened screw to apply pressure along the interface between a component and a shaft. They transmit both axial and torsional loads well, have no backlash, and tend not to loosen under cyclic loading because the screw is isolated from the shaft. Split-hub clamps are a little larger than other attachment methods, and load transmission is through friction rather than normal forces, which would be more desirable. A static analysis of a simplified model on one half of the clamp will reveal that the normal force, F_N, on each half of the shaft is about twice the screw tension, F_s, leading to peak axial force transmission $F_{axial} \approx 4 \cdot \mu \cdot F_s$ and peak torque transmission $\tau \approx 4 \cdot \mu \cdot F_s \cdot r_{shaft}$.

- **Set screws** apply a normal force directly to the side of a shaft. They are small and simple to manufacture, but provide a weaker and less robust connection than split-hub clamps. The normal forces generated by a set screw are lower both because it has no mechanical advantage ($F_N = F_s$) and because the drive feature is smaller and more delicate. Set screws tend to loosen under cyclic loading, as small variations in the geometry of the tip of the screw and the shaft lead to counterclockwise torques during some loading conditions. With very little loosening rotation, the normal force goes to zero and the connected part slips. Thread locking compounds can help reduce this issue, and are recommended. Manufacturing a shallow, flat cut, or 'flat', on the surface of the shaft greatly improves robustness of the connection and is strongly recommended.

- **Keyways and keys** are slots machined into a shaft and mating part, and tight-fitting inserts into those slots, respectively. Keyways allow the highest torque transmission, since interference between the key and keyway leads to normal loading for torque transmission. Keyways are difficult to machine, however, especially for small shafts and hubs. Without very precise manufacturing, keyways tend to exhibit backlash, in which the key is momentarily not in contact with either side of the keyway during changes in direction of the driving torque, and small amounts of slipping occur. Keys are generally not used to transmit axial loads.

- **Retaining rings** are thin metal arcs that fit into a mating groove to transmit axial loads. External retaining rings fit into grooves on a shaft, while internal retaining rings fit into grooves in the hole of the receiving part. Retaining rings generally do not provide a useful function without this groove, and are installed or removed with specialized pliers. Retaining rings otherwise have similar strengths and weaknesses to keys as discussed above.

8.3.2 Selecting Shafting

First, or in parallel, select bearings to support the shaft (see the prior section) and note their inner diameter. Next, perform a free body diagram analysis of the shaft and determine the peak internal moment and peak torque. For one candidate material, such as hardened 440C stainless steel, determine the minimum shaft diameter for each loading condition. The largest of these three diameters is the minimum diameter for your application. If the bearing diameter is largest, as is frequently the case, you can use a weaker material for the shaft, such as unhardened steel for easier manufacturing or aluminum for lower mass. Begin the catalog selection process by specifying a desired outer diameter, then work from available materials. If, instead, the larger of the minimum shaft diameters is larger than the bearing diameter, you can try to find a stronger material for the shaft, but may need to increase the size of the bearings. Since shafting comes in a discrete set of sizes, you will need to select the next-largest available diameter, making sure this is consistent with all elements interacting with the shaft. You will also need to purchase a stock length sufficient to cut the final shaft to the desired length.

8.4 Gears

Gear sets are used to constrain the angles of connected shafts or components, increase or decrease torque or speed, and alter the speed-torque relationship across transmissions. A common application is to convert the low-torque, high-speed output of an electric motor into high-torque, low-speed output at a robotic joint. (A detailed mathematical description of the relationships between gear ratios, torque and velocity can be found in the previous chapter.)

8.4.1 Spur Gears

Spur gears are a common, basic type of gear, comprising a cylindrical disc with teeth cut into the outer surface. The teeth of two spur gears interlock, or *mesh*, such that they rotate together without slipping. In selecting a spur gear set, the following terminology will be useful:

- **Pitch diameter**, D, is the diameter of the *pitch circle* passing through all the teeth in a gear at about their midpoint. Two gears will mesh correctly if they are located such that their pitch circles intersect at one tangent point, i.e. their axes are separated by a distance of ${}^1\!/_2 \cdot D_1 + {}^1\!/_2 \cdot D_2$.

- **Gear ratio**, R, is the ratio of the torque on the output gear to the torque on the input gear in a *gear set*. It is equal to the ratio of the pitch diameters, $R = {}^{D_{out}}\!/_{D_{in}}$, and commonly reported as R:1, or 'an R to one gear ratio'.

- **Diametral pitch**, P, is the ratio of the number of teeth, N, to the pitch diameter, D, in units of teeth per inch. (It is *not* the same as pitch diameter).

- **Face width**, w, is width of the face of the teeth, i.e. the size of the spur gear in the axial direction.

Left to right: Large, high-strength spur gears, high precision spur gears with split-hub (fairloc) clamps, and spur gear stock.

When selecting spur gears, the most important properties are ratio and strength, followed by size, mass, and connection style. The gear ratio is straightforward, as defined above. Gear strength requires more complex analysis, however, due to complex tooth geometry, stress concentrations, and the presence of cyclic loading. There is a rich literature on mechanical analysis of gear strength (as well as friction, vibration, and other outcomes) that we will not repeat here. Instead, the following provides a simplified, approximate analysis of small spur gears for robotics applications and equations to guide their selection.

The maximum nominal stress in the teeth of a spur gear occurs at the tooth base, or *root*, due to bending loads induced by contact with the meshing teeth. In spur gears with typical geometry, we have the following empirically-derived formula:

$$\sigma_{nom} = \frac{F_t \cdot P}{w \cdot Y} \tag{8.4}$$

where F_t is transmission load in pounds force, P is diametral pitch in teeth per inch, w is face width in inches, and Y is the Lewis form factor, a fitting term. A conservative estimate of transmission load can be obtained by assuming all torque, T, is transmitted by contact of one tooth, such that $F_t = T/(1/2 \cdot D)$. From the definition of diametral pitch, we have that $D = N/P$. The Lewis form factor is usually found in a table as a function of N and pressure angle, but for $12 \le N \le 150$ and a typical angle, we can make the approximation that $Y \approx 1/4 \cdot (N-11)^{1/8}$. Substituting these relationships into equation 8.4, we have the maximum nominal stress in terms of three free design parameters, P, N, and w:

$$\sigma_{nom} \approx 8 \cdot \frac{P^2 \cdot T}{w \cdot N(N-11)^{1/8}} \tag{8.5}$$

This nominal stress will be concentrated by the small internal fillet at the root of the tooth. With typical gear geometry, and correcting for fatigue, we can determine (e.g. from Figures A-15-6 and 6-20 of Budynas and Nisbett [2006]) that the fatigue stress concentration factor $K_f \approx 1.6$. The maximum stress is therefore:

$$\sigma_{max} = K_f \cdot \sigma_{nom} \approx 12.8 \cdot \frac{P^2 \cdot T}{w \cdot N(N-11)^{1/8}} \tag{8.6}$$

We can see from this relationship that diametral pitch has the greatest effect on stress. This makes sense, since the tooth is in bending and the diametral pitch is closely related to the thickness of the tooth at its root relative to its length, analogous to the height of a cantilevered beam relative to its length. Note also that we typically have only a few discrete options for P, meaning N and w will vary more widely across designs. We will return to this fact momentarily.

Next, we will determine maximum allowable stress. Spur gear teeth experience cyclic loading, so fatigue is an important failure mode. The endurance strength, S_e, of the gear material, or the stress at which fatigue failure will not occur, therefore provides a conservative limit. In steels, the uncorrected endurance strength is approximately one half the ultimate strength, or $S'_e \approx 1/2 \cdot S_u$, with the limit that $S'_e \leq 100$ ksi. Gears are often made from hardened steel for its strength and wear resistance, which often has $S_u \geq 200$ ksi. It is therefore reasonable to estimate S'_e as 100 ksi if the gears are hardened steel and no other information is available. If the manufacturer lists the *hardness* of the material, ultimate strength can be approximated from Rockwell C Hardness, H_{RC}, as:

$$S_u \approx 2.8 \cdot H_{RC} - 0.061 \cdot H_{RC}^2 + 0.0016 \cdot H_{RC}^3 \tag{8.7}$$

For aluminum, plastic and other materials, an appropriate substitution for the uncorrected endurance strength should be made. For example, the $5x10^8$ cycle fatigue strength for 7075-T6 Aluminum is 23 ksi, and the 10^7 cycle flexural fatigue strength for Delrin is 4 ksi.

The endurance strength, S_e, must be corrected for manufacturing, loading and environmental factors. A detailed explanation can be found in, e.g., Chapter 6 and Appendix A-15 of Budynas and Nisbett [2006]. For a gear with a machined surface, on the scale of one inch, with teeth loaded in bending, operating at room temperature, with 90% expected reliability, and applying a correction for cyclic one-way bending rather than full reversal, we have:

$$S_e \approx 0.8 \cdot S'_e \tag{8.8}$$

In a well-designed gear, the maximum stress will equal the maximum allowable stress. Combining equations 8.6 and 8.8, including a factor of safety, fos, and rearranging terms we have:

$$w \approx 16 \cdot T \cdot \frac{fos}{S'_e} \cdot \frac{P^2}{N(N-11)^{1/8}} \tag{8.9}$$

Finally, let us consider the mass of the gear. Gear mass will simply be the product of density and volume, or $m = \rho \cdot w \cdot \pi/4 \cdot D^2$. Recalling that $D = N/P$ and substituting the value of w above, we have:

$$m = 4\pi \cdot fos \cdot T \cdot \frac{\rho}{S'_e} \cdot \frac{N}{(N-11)^{1/8}} \approx 4\pi \cdot fos \cdot T \cdot \frac{\rho}{S'_e} \cdot N^{7/8} \tag{8.10}$$

Gear mass is thus affected by three separable terms: i) loading: the applied torque times factor of safety, ii) material: the ratio of the density to endurance strength, and iii) geometry: the number of teeth raised to a power near unity. Some common gear materials (with values in English units) include hardened steel ($\rho/S_e' \approx 2.6 \times 10^{-6}$), 2024-T4 Aluminum ($\rho/S_e' \approx 5.0 \times 10^{-6}$), 303 stainless steel ($\rho/S_e' \approx 8 \times 10^{-6}$), Delrin ($\rho/S_e' \approx 12.8 \times 10^{-6}$) and brass ($\rho/S_e' \approx 14.3 \times 10^{-6}$). These values should be used with caution since factors like heat treatment have a strong effect on endurance strength. The minimum number of teeth will be limited by meshing geometry (for small gears) and the desired ratio (for the larger in the set). The minimum number of teeth is also often different for different materials.

8.4.2 Selecting Spur Gears

Start by designing the smaller of the gears in the set. Equation 8.10 suggests we will typically desire a gear material with a small value of ρ/S_e', such as hardened steel. Next select the smallest number of teeth (to minimize mass and size) and the smallest diametral pitch (to minimize width), then solve for the required face width using equation 8.9. If the resulting value of w is very small compared to the values for available components, a weaker, less dense material can be chosen for reduced mass or a finer diametral pitch can be chosen to reduce the minimum diameter, improve precision, and reduce friction and vibrations. If the desired w is larger than available, a different source must be found or the number of teeth must be increased; for a set pitch the number of teeth is proportional to the diameter, such that increasing N reduces tooth load for a given torque. Finally, select a means of connecting the gear to the shaft or component. Split-hub clamps are an excellent choice in most gear designs since they precisely center the gear on the shaft, have zero backlash, do not loosen under cyclic loading or vibration, and have high torque transmission compared to, e.g., set screws. Iterate through the above steps until a satisfactory smaller gear has been determined.

Next, select the number of teeth in the larger gear. This should be, approximately, the product of the number of teeth in the smaller gear and the desired gear ratio, $N_l = R \cdot N_s$. If the desired number of teeth is not available, you might try selecting a different small gear. If the desired number of teeth far exceeds the available gear sizes, you may need a multi-stage gear reduction, i.e. two gear sets in series. The larger gear must match the smaller gear in all aspects other than diameter and number of teeth, including diametral pitch (or the gears will not mesh), and face width and material (or the teeth of the larger gear may fail). The larger gear may, however, have internal features to reduce mass and inertia, such as pockets or holes, depending on the size and torque.

8.4.3 Bevel Gears, Worm Gears and Rack and Pinion Gears

With properly shaped and oriented teeth, gears need not have parallel axes nor lie in the same plane. Spur gears are a special case of this more general set, which also includes bevel, worm and helical gears:

- **Helical gears** have teeth oriented at an angle relative to the primary direction of load transfer, making them helical in shape and causing them to mesh more smoothly, reducing vibrations at high speeds. The primary types are *parallel*, which are nearly identical to spur gears save this change, and *tranverse*, which are similar to worm gears (see below).

- **Bevel gears** are conical, rather than cylindrical, allowing the axes of the connected shafts to intersect at some angle, typically 90°, rather than being parallel. Bevel gears are sometimes used without a ratio, simply to allow rotation in a different plane. Bevel gears require additional bearing surfaces since the pressure angle creates axial as well as radial loads on the supporting shaft (see below). Shafts supporting bevel gears cannot physically intersect, meaning that they are often loaded in cantilever, which increases bearing loads and makes alignment and stiffness more challenging.

- **Worm gears** look and operate like a lead screw meshed with a spur gear, except for having teeth that mesh appropriately. They allow the driving axis to be oriented at an angle with the driven axis, typically 90°, with non-intersecting axes, typically with the driving gear nearly tangent to the pitch circle of the driven gear. Worm gears usually have a large gear ratio, and are usually non-backdrivable (see below).

- **Rack and pinion** gears have one round rotating gear, the *pinion*, meshed with one straight linearly translating gear, the *rack*. They are similar to a spur gear set where one gear has infinite diameter, or to a worm gear where the worm element translates rather than spinning.

Left to right: Steel helical bevel gears with a one to one ratio, plastic bevel gears with a higher ratio, and a steel worm gear.

Nearly any relative orientation of gears can be achieved with the correct tooth shapes if needed for a unique application. In some configurations, the meaning of 'gear ratio' can become confusing. In some of these cases, the ratio can be obtained by comparing the number of rotations the input shaft needs to make to obtain one full rotation of the output shaft. In all cases, careful static analysis using free body diagrams will provide an answer.

8.4.4 Gear Integration

Alignment precision and stiffness are critical if gears are to mesh correctly. If axes are too far apart, or are pushed apart due to compliance, this will cause backlash, skipping of teeth, or increased tooth moments and breakage. If the axes are too close or not parallel (or whatever the desired alignment angle), this will increase radial bearing loads, friction, and wear. In any assembly involving gears, it is therefore important to consider stiffness, tolerances and error when laying out component connections and when selecting shafting and (ball) bearings.

Reaction forces and moments from a gear onto its driving shaft include not only the primary transmitted torque, but also a combination of radial and axial forces and moments due to the way their teeth interact. The *pressure angle* is the angle of the normal force on the tooth with respect to a tangent line, about 20° for a typical gear. A spur gear with pitch radius r and driving torque T will therefore have a force $F_{tan} = T/r$ in the tangent direction and $F_{rad} = T/r \cdot \sin(20^\circ) \approx T/(3 \cdot r)$ in the radial direction. In bevel gears, this push-out force will have both radial and axial components, each with magnitude $\sqrt{2}/3 \cdot T/r$ for a 90° bevel gear set. Since the axial component is applied at the gear tooth, it will also cause a moment at the shaft connection on an axis orthogonal to primary torque axis, with magnitude $M_\perp \approx \sqrt{2}/3 \cdot T$ for a 90° bevel gear set. These loads must be accounted for when designing supporting structures.

A gear set (or box) is *backdrivable* if the largest gear may be rotated to turn the smaller gear(s). It is always possible to apply a torque to the smaller gear in a set and cause the larger gear to rotate (assuming it is unopposed), but the opposite is not true. As we found in our analysis of screw tightening, the effects of friction depend on which element in the sliding interaction is driving the motion. If the coefficient of friction and gear ratio are high enough, applying a force to the larger gear will simply bind the gear train and prevent rotation. This is analogous to the difference between kicking the narrow edge of a door wedge and stepping down on its top face. As a heuristic, gearboxes with 50% or lower efficiency and worm gears tend not to be backdrivable.

8.5 Timing Belts and Pulleys

Timing belts and pulleys are similar to spur gears, with the difference that pulleys interact through a belt rather than meshing directly. This has the advantage that small pulleys can be located at a distance, providing the belt has the correct length to wrap over them precisely.

Timing belts are rated for driving tension, equivalent to the tooth force in spur gears, or the driving torque divided by the pitch radius. The peak driving tension is typically linearly proportional to belt width. Exceeding the driving tension will cause the polymer teeth of the belt to slip past the teeth in the timing pulley, or wear the belt teeth quickly, or shear the belt teeth.

Timing belts require some amount of *pretensioning* to prevent slippage as they stretch under load. Pretensioning can be achieved through residual loads applied during assembly, e.g. forcing a supporting component sideways and then tightening it down, or through an additional *idler pulley* that applies a force perpendicular to the belt in an unsupported region. Pretensioning elements can also help take up slack arising from imprecise mounting.

The need for pretensioning, and problems arising from loose belts, make assembly of timing belt drives tricky. Trying to slide a set of pulleys onto a shaft while holding the belt in tension will usually cause binding and is ill advised.

Some belts will tolerate bending along the belt axis, but timing belts work best if the two pulleys lie in the same plane.

A variety of timing pulleys, timing belts, and idlers.

8.6 Steel and Synthetic Cables

Cable, also known as rope or wire rope, can be used to provide static tensile forces or to transmit power. When used for power transmission, the elements onto which the rope wraps are called *capstans* or *drums*, elements for rerouting rope are *pulleys*, and the overall design is a *cable drive* transmission. The most important properties to consider when selecting rope are breaking strength and flexibility. Breaking strength is most strongly affected by material and diameter. Strength is measured under ideal conditions and should always be used with a large factor of safety, especially if the rope is cyclically wrapped and unwrapped. Flexibility of a cable is determined by its material and its *construction*, or the weave of the individual fibers. Small cables with many small fibers and small sub-weaves tend to result in more flexible construction, but lower strength and higher cost.

Common cable materials include stainless steel and polymers like Vectran. Stainless steel is stiff and strong, but tends to be relatively resistant to bending and is susceptible to abrasion, requiring larger diameters in well-designed cable drives. As a heuristic rule, the minimum capstan or pulley diameter is 20 times the diameter of the wire rope. Coatings help prevent wear and improve gripping for wire ropes. Vectran is strong, flexible and abrasion resistant, but also exhibits more *stretch*, or elongation under load, and *creep*, or permanent changes in nominal length, than steel ropes, making it less desirable for high-precision drives such as in haptic interfaces. Vectran can be wound on drums as small as 3 times the rope diameter and will hold knots. Nylon, monofilament fishing line, natural-fiber rope and other polymers usually have poor performance in robotics applications.

8.7 Springs

Springs provide force as a function of displacement, which can be used to provide passive loading, store energy, or even to add a degree of freedom as a *flexure*. The most important properties in spring selection are peak force and stiffness (or, equivalently, displacement).

Minimum spring size is strongly influenced by the energy stored, spring geometry and material properties. The most efficient spring geometry is a prismatic member in axial load, since each element experiences equal (maximal) stress and strain, but most engineering materials are too stiff to make this layout practical. Helical coil springs are 3 times heavier than this standard, because the wire is loaded in torsion, and are made from steel which has relatively poor energy storage density (see below), but are compact and widely available. Leaf springs are 9 times heavier than the axial standard, because they experience cantilever bending loads, but can be constructed from composite materials with superior energy storage density.

For most spring geometries, the ideal spring material will minimize $\rho E \sigma_y^{-2}$, where ρ is density, E is elastic modulus, and σ_y is yield strength. Best in class materials include unidirectional fiberglass used in leaf springs, $\rho E \sigma_y^{-2} \approx 6 \times 10^{-5}$, music wire (hardened steel) used in helical coil springs, $\rho E \sigma_y^{-2} \approx 5 \times 10^{-4}$, and synthetic rubbers used in axial springs, $\rho E \sigma_y^{-2} \approx 5 \times 10^{-5}$. While the energy storage density of rubber is comparable to fiberglass and better than steel, rubber also has nonlinear force-displacement characteristics, including direction-dependence (hysteresis) and rate dependence (damping and relaxation), which can be problematic in some applications.

To select coil springs, first determine the required force and displacement from static and geometric analysis. Next, consult a catalog and narrow the options by peak force and then by deflection or stiffness. Novice designers are frequently surprised by how large a spring must be to deliver the required force and displacement, so be prepared to make significant design modifications to fit a long resting length or reduce energy storage requirements. To design axial rubber springs or fiberglass leaf springs, use simplified models of stress and displacement to select optimal geometric parameters, such as length, cross-sectional area, width or height, using the inverse-analysis methods detailed in Chapter 5.

8.8 Prominent Catalog Sources

There are many companies that sell catalog components, and many more still that specialize in supplying a particular type of component to both catalogs and industry. Here is a sample related to course projects, research, and initial prototypes, including some good places to look first:

- **McMaster-Carr**, `http://www.mcmaster.com`, is a good place to look for fasteners, bearings, shafting, cable, and springs. They offer a wide variety of other machine components, with a bias towards larger applications. McMaster has fast shipping and a user-friendly website. Watch out for shipping costs, which are not predicted during checkout.

- **Stock Drive Products**, or Sterling Instruments, `http://www.sdp-si.com`, is a good place to look for small-scale gears, timing belts, pulleys, chains and sprockets. Their web interface is also reasonably user friendly.

- **MSC Industrial Supply**, `http://www1.mscdirect.com`, has similar offerings as McMaster-Carr.
- **W. M. Berg**, `http://www.wmberg.com`, has similar offerings as Stock Drive.
- **Quality Transmission**, `http://www.qtgears.com`, has similar offerings as Stock Drive.
- **Dragon Plate**, `http://www.dragonplate.com`, sells a variety of carbon fiber stock and offers custom machining.
- **Carbon Fiber Tube Shop**, `http://www.carbonfibertubeshop.com`, sells... carbon fiber tubes.
- **Gordon Composites**, `http://www.gordoncomposites.com`, offers high-quality composite materials well-suited to leaf springs.
- **West Marine**, `http://www.westmarine.com`, sells synthetic ropes such as Vectran. And sailing equipment, if that's handy.
- **Harmonic Drive**, `http://www.harmonicdrive.net`, specializes in compact, high-ratio gear sets.
- **Airpot**, `http://www.airpot.com`, sells high-quality dashpots and dampers.

8.9 Acknowledgments

Thanks to Kirby Witte for help in editing this chapter.

Bibliography

Richard Budynas and Keith Nisbett. *Shigley's Mechanical Engineering Design*. McGraw-Hill, 8th edition, 2006. ISBN 0-390-76487-6.