Engineering Design I: Methods and Skills

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Chapter 2

Simple Models

2.1 Modeling

Modeling is the process of generating abstract, conceptual or mathematical representations of complex, real phenomena. You have often used models, explicitly or implicitly, in other courses. For example, you may have used rigid body dynamics models to calculate angular accelerations in Dynamics, or you may have used an Euler-Bernoulli model of beam stress to calculate the peak stress in an I-beam during Stress Analysis. These abstractions are necessarily simpler than the real systems they represent, neglecting, for example, gravitational or anisotropic effects, to allow for deeper consideration of the most important, and basic, system features. Whereas in prior courses you may have been provided with problems that have already been reduced to model form, in Design I you will often need to develop your own models of mechanical systems that you are in the process of designing. When done well, these simplifications will allow for the application of analytical tools you have learned in prior courses, or new analytical tools relevant to the system of interest, which can be easily turned into powerful tools for optimizing a mechanical design. Thorough consideration of these basic abstractions will ensure that more detailed designs are built upon sound foundations.

2.1.1 How to generate models

Useful models are as simple as possible while remaining relevant, allow use of analytical methods, and build on prior modeling efforts where possible. Modeling can often be a messy, creative, and iterative process, but the following principles can help guide you:

- *As simple as possible, but no simpler.* Einstein – Try to use the simplest representation of the design that is capable of producing the phenomena of interest. For example, load analysis might only require limited geometric representations, such as lever arm lengths, while stress analysis might require cross-sectional areas.

  – *Parameters = complexity.* One indicator of model complexity is the minimum number of parameters required to fully describe the model. For example, a lever could be modeled as a rectangular extrusion, with length, width and height, but would more simply be modeled as a line, with only length, if suitable for the application. On the other end of the spectrum, a mesh model of a solid body used for Finite Element Analysis could have thousands of geometric parameters.

- *Reduce complexity to allow access to analytical tools.* A simplified representation is only useful if the simplifications allow application of more rigorous design or analysis tools. As an example, simplifying an octagonal pipe as septagonal would not be very helpful, due to a lack of septagonal flow models. Simplifying it to a circular pipe, by contrast, would allow application of powerful mathematical tools. Your training as a mechanical engineer has made you aware of many such models (which we will not duplicate here) and you can also find new, relevant models when beginning a design in an unfamiliar domain.

- *Address current design goals.* The same system might be modeled in different ways at different stages of the design process. For instance, you might initially model a lever as a line (with just one or two length parameters) during load analysis. Later, you might model the lever as a rectangular extrusion (with two additional parameters and finite cross-section) to allow simple stress analyses.

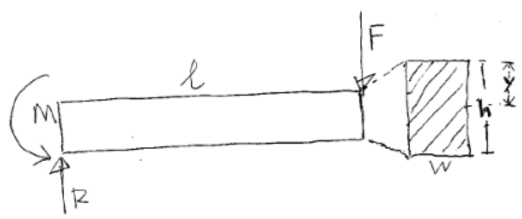
- *Build up complexity.* Start with the most basic model that captures features of interest and really, truly, deeply understand the significance of these abstractions first. For example, based on load analysis alone, you might see that certain geometric parameters have a strong effect on peak forces experienced by a component. Then, add complexity as you smoothly transition through models of increasing complexity to a detailed design. You may find that you need to return to simpler models to provide additional information at more complex stages, e.g. additional load analysis to inform stress analysis of more complex designs.

- *Keep it symbolic.* As you develop a mathematical model addressing an outcome of interest, such as peak stress, it will be tempting to immediately substitute in numerical values for known design requirements or constraints.

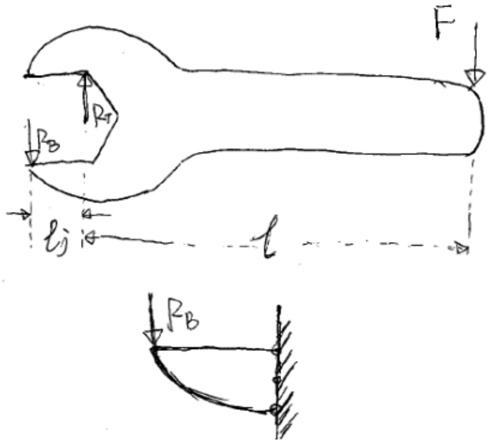
You may even be tempted to guess as to the best values for some design parameters. Resist this temptation! Fundamental relationships are easier to observe in symbolic equations. Equations with long decimal numbers are more cumbersome to manipulate and often mask the cancellation of design variables. Until you need numerical values, e.g. to verify feasibility or to construct a CAD model, keep your equations in symbolic form.

Modeling Example: Adjustable Wrench

In this example, let us consider simple models of a wrench. We will address different outcomes and slowly build from simple to complex representations.

Application	**Model**	**Equations**
Load analysis (FBD)		$F = R$ $M = F \cdot l$
First-order stress analysis		$\sigma_{max} = \frac{M \cdot y}{I}$ $\sigma_{max} = \frac{6 \cdot F \cdot l}{w \cdot h^2}$

Jaw Stresses

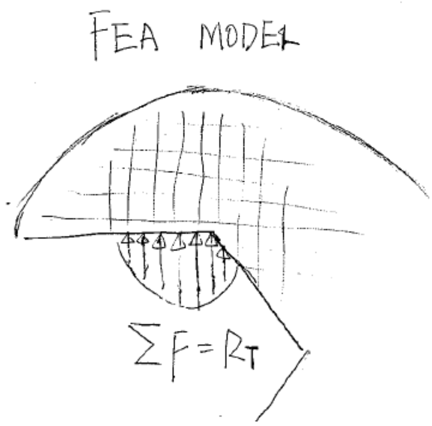


$$R_B = \frac{l}{l_j} F$$

$$R_T = F + R_B$$

$$\sigma_{max} = \frac{R_B \cdot l_j \cdot y_j}{I_j}$$

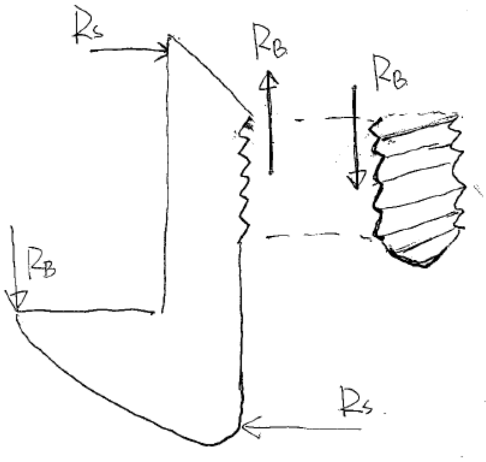
Contact
Stresses



$$\int p = R_T$$

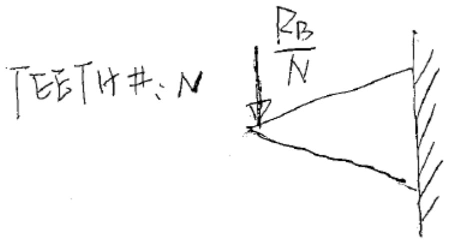
many coupled
linear eqns...

Adjustment
screw load



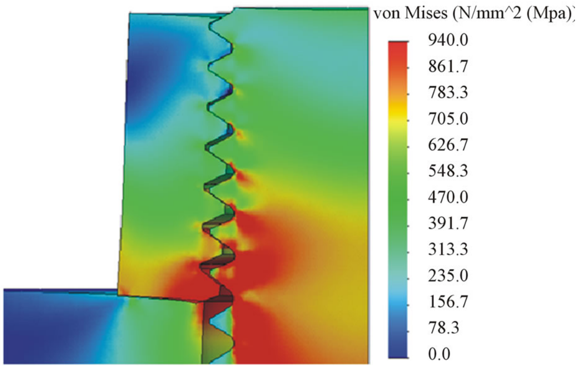
$$M_{RS} + M_{RB} = 0$$

Adjustment screw stress



$$\sigma_{max} = \frac{\frac{R_B}{N} l y}{I}$$

Finite Element Analysis of adjustment screw stress



many coupled linear eqns.

2.1.2 A Brief Side-Note on Units:

In prior courses, your professors have told you that SI units (Système International: meter, kilogram, second) are superior to English units (inch, pound, second). For a variety of reasons, they were correct. The power of inertia, however, is not to be underestimated. In the U.S., most mechanical engineering design components are still produced to English standards. In fact, your humble professor learned design using English units, and has intuition based around these numbers. Therefore, we will use English units for nearly everything in this course.

	Length	Mass	Force	Energy	Power
SI	m	kg	$N = kg \cdot m/s^2$	$J = N \cdot m$	$W = N \cdot m/s$
USC	ft	lb	lb	$ft \cdot lb$	hp

Chapter 3

Free Body Diagrams

Free Body Diagrams (or FBDs) are a type of simple model used for basic mechanics analysis. In this course, we will primarily consider (quasi-) static systems, in which FBDs can be used to calculate the loads experienced by mechanical components, features of those components, or assemblies of multiple components. Although we have all performed many static load analyses using FBDs, mastery of the technique is needed for mechanical design; you will consider many new, potentially-confusing, and rapidly changing scenarios as you iteratively improve a design. Deciding where to draw boundaries and which loads are important can also be tricky. So, let's review:

3.1 How to Perform Static Load Analysis with FBDs

The following steps and principles will help you to generate useful free body diagrams during the design process. With practice, you will be able to perform this analysis quickly and intuitively. However, we have found that many students' confidence outpaces their intuition in this area, so be rigorous, especially when beginning a new design problem.

- *Define a *Free* body.* Free body diagrams begin with a free body. Seems obvious, right? But in the heat of a concept design brainstorming session, the lines can get blurred. Be sure you have clearly defined what stuff is part of the "free body" and what isn't, i.e. clearly define the imaginary boundary separating the system from the environment. To best avoid confusion, re-draw the free body in isolation, without any of the connecting pieces. Remember that you can define a free body any way you want. For example, you could draw the boundary of your free body around a small part of a single component, or around an entire assembly with lots of parts. Just make sure you are consistent about what's inside and outside the boundaries.

- *Identify all possible external loads.* Newton tells us that the way physical objects interact is through forces. The way that your free body knows it is part of some bigger world is *only* because that world exerts forces on it at its boundaries. Newton also tells us that equivalent loads (forces or moments) on a free body will lead to the same behaviors, regardless of the source. So, from the perspective of your free body, being inside a broader assembly is equivalent to being acted upon by the forces and moments you draw in your diagram. When describing the external loads on your system, keep in mind:

  – *All external loads.* External loads generally occur at boundaries, although gravitational or magnetic forces act within boundaries. Recall the "Rule of joint resistance": If a joint at the boundary of a free body could resist a translational displacement along or rotational displacement about a certain axis, then a corresponding force or moment must be represented on the free-body diagram.

  – *No internal loads.* Any forces between elements inside the free body should not be included as external forces. All rigid bodies are jam-packed with internal forces, which could be seen by generating new free body diagrams at an infinite number of boundary locations. These are all equal and opposite, however, and therefore cancel when considering the free body as a whole.

  – *Ignore negligible loads.* Some external loads, while strictly valid, will not contribute significantly to the overall part loading, but will complicate load analysis equations. In particular, gravity can usually be neglected for small parts that undergo large external loads, while friction can often be neglected at intentional degrees of freedom, such as bearings.

- *Perform force and moment balance.* Newton's second law tells us that the rate of change of momentum in a system is proportional to the external forces and moments, commonly stated as $\Sigma F = m \cdot a$ and $\Sigma M = \dot{H}$ (where H is the angular momentum of the system). It is often useful during mechanical design to treat a system as static or quasi-static, such that $a \approx 0$ and $\dot{H} \approx 0$, and the key equations reduce to $\Sigma F = 0$ and $\Sigma M = 0$. These sets of equations, commonly referred to as force and moment balance, can be applied to derive relationships between known forces, geometric design parameters, and unknown (reaction) forces in a free body diagram. In a planar model, there are two force balance equations, e.g. $\Sigma F_x = 0$ and $\Sigma F_y = 0$, and one moment balance equation, e.g. $\Sigma M_z = 0$, while in a three-dimensional model there are six equations.

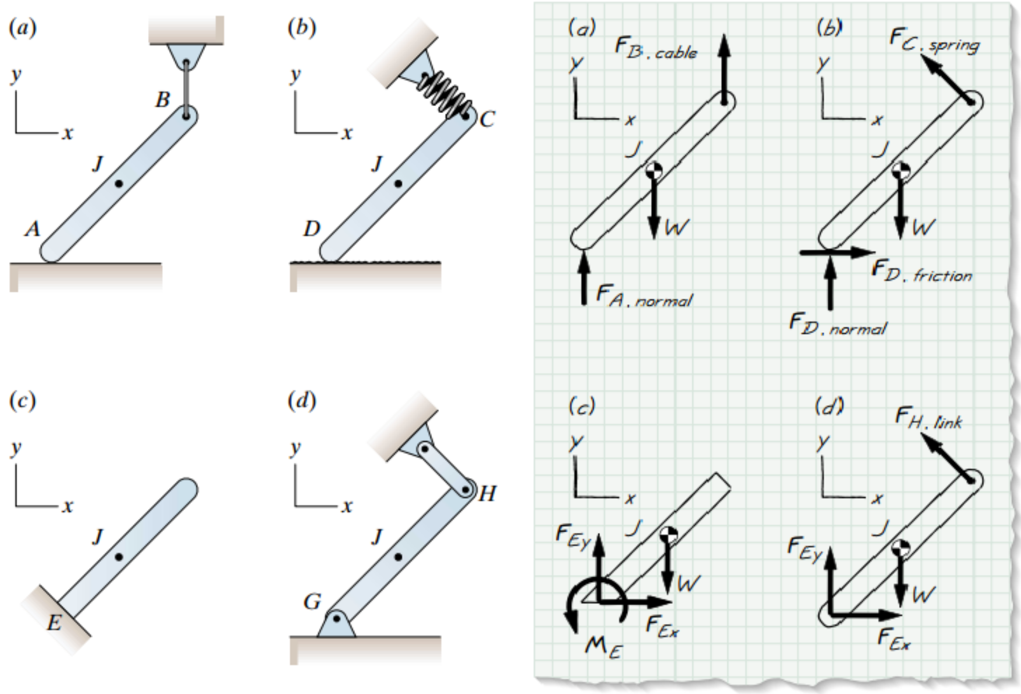
- *Under-constrained FBDs.* If you find more free load variables (reaction forces and moments) than equations, the system is said to be 'under-constrained' or 'statically indeterminate'. That is, there are infinite possible solutions that all satisfy static equilibrium. *What to do:* First, make sure you have FBDs for all the interesting boundaries; one FBD will often reveal components of force needed to resolve others. Second, try making simplifying assumptions, e.g. about the direction of action of external forces or whether some forces or moments can be neglected. Be sure to check that your simplifying assumptions are theoretically valid, for example they should still allow for static equilibrium to be possible. Third, consider the compliance of the free body. Loads that would require large displacements to become significant are likely to play a smaller role than those that engage at small displacements. Try imagining how the parts would squish when loaded. This might be easier to envision if you imagine the parts were made out of an elastic material, such as rubber or even Jello. Formally modeling the body as compliant can generate additional equations, as we learned in Stress Analysis, relating reaction forces to one another through strain or displacement and adding needed constraints to the solution space. This will provide an analytical solution (see the accompanying worked examples) although in many cases the analysis is laborious, limiting its use in our fast-paced hand analysis activities. Alternatively, you can check your guesses using finite-element analysis of CAD mock-ups (see below). Fourth, think about ways to design the system to avoid this problem; to understand that a design is good, it helps to design it to be easily understood.

- *Over-constrained FBDs.* Similarly, it is possible that solutions do not generally exist for your system, i.e. there are more equations than unknown variables (such as reaction force and moments). *What to do:* You have either neglected important forces or moments, or your design is not in static equilibrium. Go back to the boundaries and make sure you've included all reactions. If you have not missed any, this component will move when loaded...

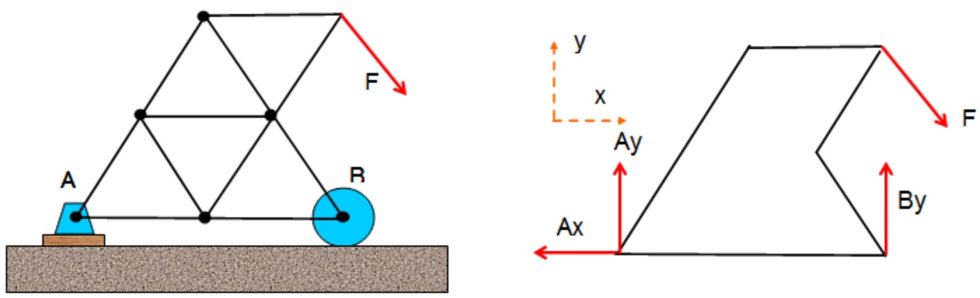
- *Using CAD Mock-Ups.* When dealing with tricky statics problems, it may be useful to check your assumptions using finite-element analysis of mock-ups in CAD software. For this purpose, you don't need a complete and accurate model of the design, which could take a long time to construct and analyze. Instead, you just need something that captures the main features of the part you're interested in. Be sure that the (known) loading and boundary conditions are as close as possible to what you expect in the real design. (For example, fixing the inside of a hole will allow moments to be generated, whereas a bearing constraint would not, which could strongly affect your result.) Then run the analysis to see what happens. If the deformed shape and pattern of stress are consistent with your assumed reaction forces, this is a good sign. If not, try to interpret the numerical result in terms of alternate simple models. For example, if the stress pattern suggests bending, be sure that your guessed reaction forces include a component of force orthogonal to the feature that is in bending. You can iteratively guess at loading conditions, check by analyzing a simplified CAD mock-up and change assumptions in your FBDs until you have a good understanding of part loading. Later, as we begin to design components, you can add the step of iteratively improving the shape of your design to remove undesirable loading types or accommodate the loading conditions that do arise. In this case, you would iterate until you understand your design, the loading conditions are favorable, and the shape of the part is well-suited to those loading conditions.

Consider the below FBD examples. Check that you understand which forces were included, why, and whether the reaction loads can be uniquely resolved using static balance.

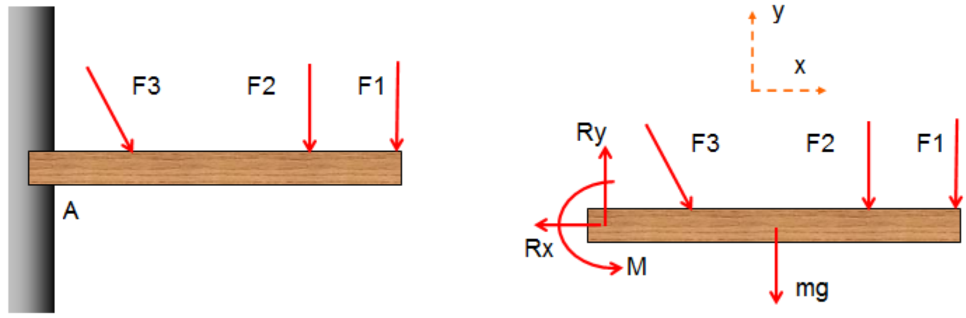
Example 1: (Sheppard and Tongue, 2007) Each system consists of a uniform bar of weight W, supported in different ways.
a: Normal contact without friction at A, cable attached to the system at B.
b: Spring attached to the system at C, normal contact with friction at D.
c: Fixed to its surroundings at E.
d: Pinned at G, and a link attached to the system at H.



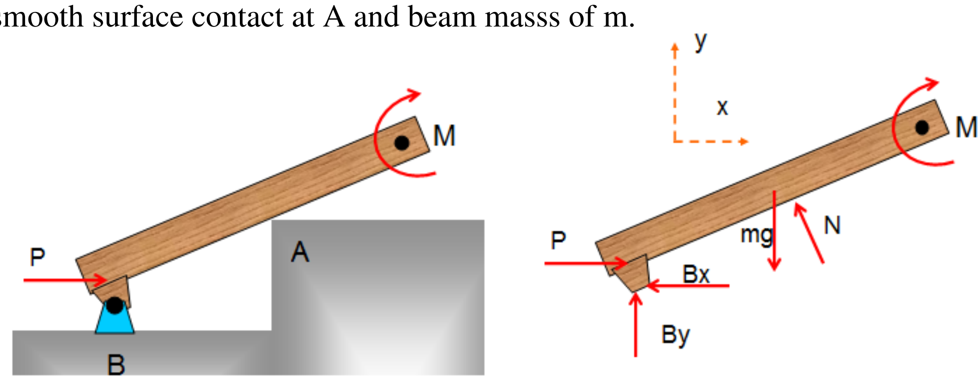
Example 2: Planar truss with negligible weight compared to force F



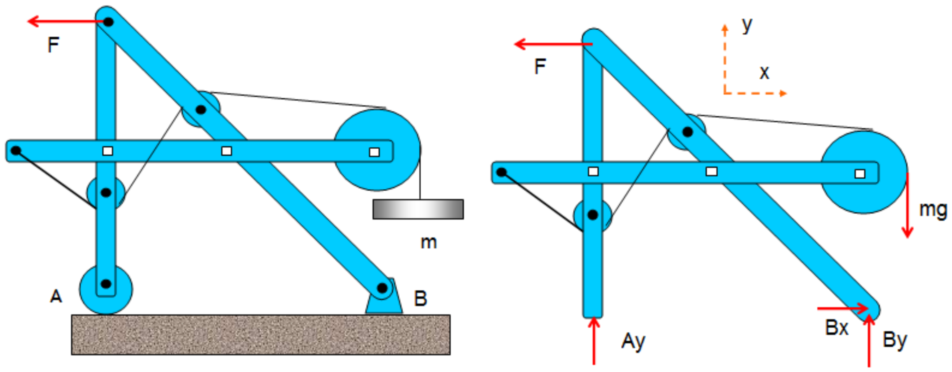
Example 3: Cantilever beam with mass m and external forces.



Example 4: One-end hinged beam on stairs with external torque. Assuming smooth surface contact at A and beam masss of m.



Example 5: Interconnected bodies with external force F and negligible system weight.



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Bibliography

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