Engineering Design I: Methods and Skills
Topic Readings

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Chapter 4

Component Failure Analysis

4.1 Mechanical Failure

Most broadly, *failure* of a mechanical component occurs when its function has been compromised beyond use. Some common examples in the design of mechanical components include:

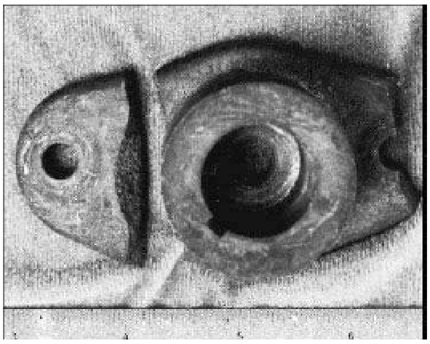
- Breaking of a part into two or more pieces (fracture);
- Permanent distortion of a part (plastic deformation);
- Reduced reliability, e.g. due to wear or the presence of cracks; and
- Unacceptable displacement of a key point (even if it is elastic).

Failure analysis therefore encompasses stress analysis, but also a wide range of additional domain-specific analyses. In this course, we will use analytical modeling approaches [covered in detail in, e.g. Budynas and Nisbett, 2006] that you have learned in prior Mechanical Engineering courses to address:

- Failure due to static stress exceeding the yield strength of a material;
- Failure due to excessive displacement;
- Failure due to buckling, a special combined case; and
- Failure due to cyclic loading, i.e. fatigue.

4.2 Failure Due to Excessive Static Stress

A *static load* is a force or moment that does not change in magnitude, point or points of application, or direction over time. A static load can produce axial tension or compression, shear, bending, torsion, or any combination of these.



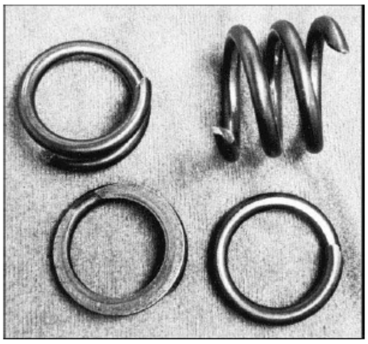


Figure 1. Examples of parts that failed under single applications of a large load. *Left:* A lawn mower blade driver hub that failed during impact with another structure. *Right:* A valve spring that failed during a surge of force in an over-sped engine. From Budynas and Nisbett [2006].

Failure due to excessive stress under static loads can be classified as:

- *Fracture*, which occurs if *ultimate stress*, or S_u, is exceeded.
- *Plastic deformation*, which occurs if *yield stress*, or S_y, is exceeded.

Ductility refers to the amount of plastic deformation that occurs prior to breakage. Ductile materials undergo significant deformation, like a piece of taffy, prior to finally breaking. Brittle materials, by contrast, undergo very little deformation before they snap. More precisely, using the *fracture strain*, ε_f, or the engineering strain at the moment of part fracture:

- *Ductile* materials exhibit $\varepsilon_f > 0.05$, and are characterized by S_y and S_u
- *Brittle* materials exhibit $\varepsilon_f < 0.05$, and are characterized only by S_u

4.2.1 General Rule: Compare S_y to σ_m or von Mises

Plastic deformation will usually constitute failure for parts you design. In this course, we will use experimentally-determined yield strengths (or stresses) of materials, denoted S_y (or σ_y). These strengths are typically determined in well-defined tests (standardized by the ASTM) in which a part is subjected to, e.g., pure axial loading (axial strength) or pure bending (flexural strength). Results are interpreted using the same simple models you learned in the prior chapter, i.e. $\sigma = F/A$ and $\sigma = My/I$, respectively. Therefore, strength values can be cleanly compared to maximum stress, σ_m, determined by hand analysis. Similarly, the von Mises equivalent stress, a common default output of FEA stress analyses, is equivalent to the hand-calculated stress values for idealized material strength experiments. If the material in question is brittle, simply substitute S_u for S_y.

4.2.2 Other Stress Failure Models

In some design domains (not in this course) you may need to have a more accurate failure model for a particular material or load scenario. Perhaps high accuracy is required, or plastic deformation is acceptable, or even desired. Perhaps the material has extreme properties, qualitatively different from the common robotics materials we will use. Perhaps the scale is very small or very large, or, if you like, very fast or very long. Where to look then? There are many models that could be applied in such situations, and some of these are covered in detail in analysis texts such as Ch. 5 of Budynas and Nisbett [2006]. We will touch on these only very briefly below. In practice, experiments with prototypes or material samples will provide the best data for accurate failure prediction.

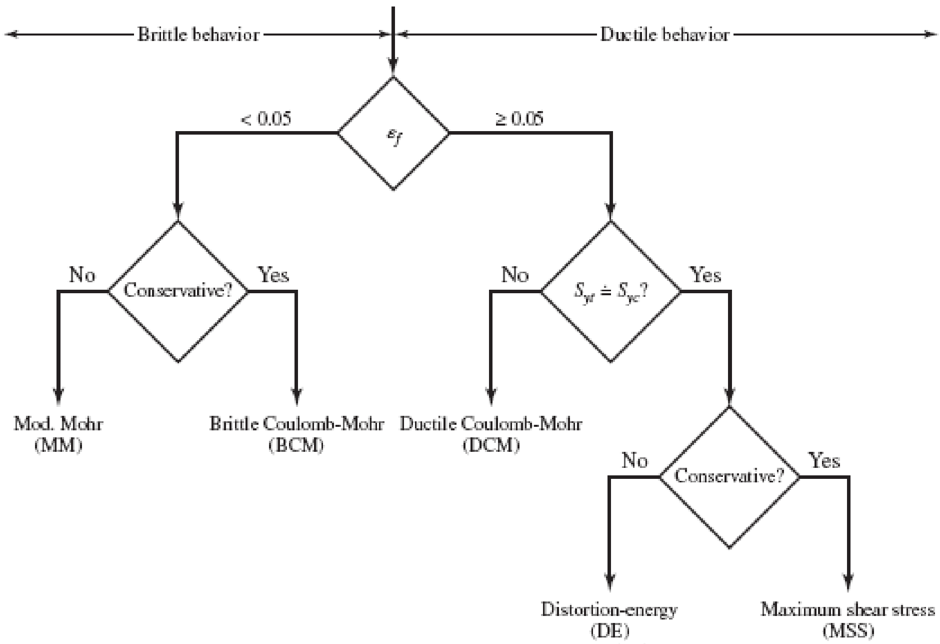


Figure 2. Failure model selection flow chart [Budynas and Nisbett, 2006]. Note that the von Mises equivalent we will use is derived from Distortion Energy theory.

Ductile materials normally have an identifiable yield strength that is often the same in compression as in tension ($S_{yt} = S_{yc} = S_y$). They exhibit significant plastic deformation before finally breaking. The microscopic mechanism of ductile failure is the deformation by motion of atomistic defects [Steif, 2012] and this type of failure depends on shear stress. Some generally accepted theories of yield criteria for ductile materials include Maximum Shear Stress (MSS), Distortion Energy (DE) and Ductile Coulomb-Mohr (DCM).

Brittle materials do not exhibit an identifiable yield strength and are typically classified by ultimate tensile and compressive strengths, S_{ut} and S_{uc} respectively. They break with no evidence of plastic deformation, like glass. The microscopic mechanism of brittle failure is the opening up and propagation of small cracks and it depends on normal stress. Generally accepted theories of fracture criteria of brittle materials include Maximum Normal Stress (MNS), Brittle Coulomb-Mohr (BCM), and Modified Mohr (MM).

4.2.3 Failure Due to Excessive Deflection

Excessive stretching of a structure, without reaching the yield stress of the material, can also make the structure unsuitable to a task. Compression, tension, torsion and bending loads can all lead to excessive deflection. Here are some common deflection modes and the corresponding simple mathematical models for calculating peak deflection.

Axial loading: $\delta = \frac{FL}{AE}$, where δ is axial endpoint displacement

Cantilevered beam: $\delta_e = \frac{FL^3}{3EI}$, where δ_e is orthogonal endpoint displacement

Torsion rod: $\theta = \frac{TL}{JG}$, where θ is angular displacement of the rod end

For each of the above, F is the applied force, L is the axial length, A is cross-sectional area, E is (tensile) elastic modulus , I is area moment of inertia (about the centroid), T is applied torque, J is polar moment of area (about the axis), and G is shear (elastic) modulus.

4.2.4 Failure Due to Buckling

A special case of *unstable* deflection leading to catastrophic failure is *buckling*. Buckling typically occurs in long, thin “columns” under compression, but can also occur in any shape of component or feature under compression. Buckling occurs because (infinitesimally) small deflections lead to increased leverage of the applied loads, resulting in even more deflection, creating a positive-feedback loop. Failure can be estimated using the Euler buckling model:

$$F_{cr} = \frac{C\pi^2 EI}{L^2}$$

where F_{cr} is the critical load, i.e. the load at which the column becomes unstable and buckling is expected, C is a boundary constraint constant (see the figure on the next page), E is the modulus of elasticity, I is the cross-sectional area moment of inertia, and L is the length of the column.

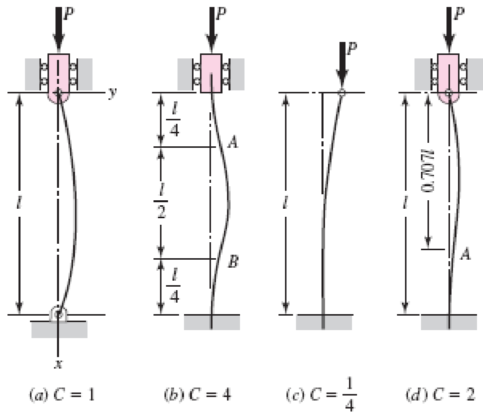


Figure 3. Buckling model boundaries and corresponding constants.

4.3 Failure Due to Fatigue under Cyclic Loading

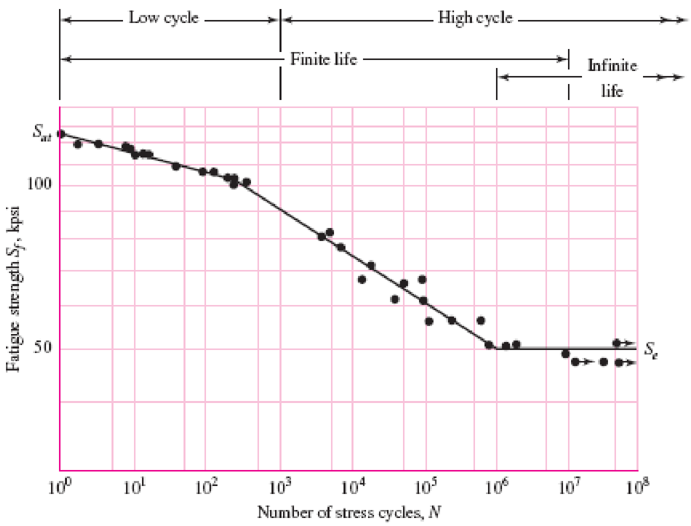
Mechanical parts frequently experience stresses that vary with time. Often, machine components are found to have failed under the action of repeated or fluctuating stresses, yet careful analysis reveals that maximum stresses were well below the yield strength. These failures usually occur after the stresses have been repeated a very large number of times. This is called *fatigue failure.*

Fatigue usually involves three stages of development (for your curiosity):

- Stage I: the initiation of one or more micro-cracks, due to cyclic plastic deformation followed by crystallographic propagation.
- Stage II: micro-cracks progress into macro-cracks, forming parallel plateau-like fracture surfaces separated by longitudinal ridges.
- Stage III: during the final stress cycle the remaining material cannot support loads, resulting in a sudden, fast fracture.

4.3.1 Fatigue Life and Strength

To determine whether a part will fail under cyclic loading, it is no longer sufficient to compare the maximum expected stress to the static yield strength of the material, S_Y. We must include information about the expected number of loading cycles, N. As the number of cycles increases the sustainable stress, S, decreases. $N = 1$ for static loading and the sustainable stress, S, is equal to S_Y. For $N > 1$, S will be less than S_Y. Fatigue analysis is often based on charts of S vs. N, for a particular material and set of conditions, an example of which is shown in Figure 4 for carbon steel.



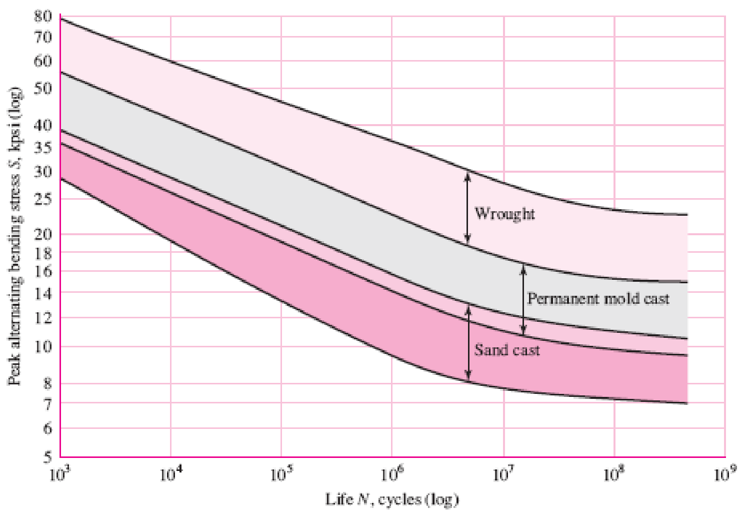


Figure 4. Examples of allowable stress (S or fatigue strength) as a function of the number of loading cycles (N). ***Top:*** One grade of carbon steel. If we expect the

component to undergo 10^4 cycles, the allowable stress would be around 70 ksi. Notice that below 50 ksi, the fatigue life (N at failure) is approximately infinite. The "endurance limit", S_e, or stress below which fatigue is not expected, for this material is therefore about 50 ksi. ***Bottom:*** An aluminum alloy, formed using various manufacturing methods. Both low surface quality and the presence of asperities accelerate the early stages of fatigue. Aluminum (and most materials other than steel) generally does not have an endurance limit, i.e. with enough cycles aluminum will typically fail. However, below 14 ksi, this grade of wrought aluminum will persist for more than 10^8 cycles, which is often used to define "fatigue strength". Reproduced from [Budynas and Nisbett, 2006].

4.3.2 Analysis of the Endurance Limit

Many materials will survive a number of load cycles, N, approaching infinity as long as the applied stress is below a maximum value known as the *endurance limit*. The endurance limit of an ideal sample, S'_e, is shown in Figure 4 Top at the point where the curve becomes horizontal approaching infinite loads (approximately 50 kpsi in this case). The ($'$) denotes that this endurance limit is for an ideal specimen only. The endurance limit, S_e of an unideal machine part can be calculated using many methods, each may be more or less accurate depending on part geometry and loading. A method is shown here for calculating the endurance limit of a cylinder in bending as an example, but it is recommended that students find other sources to determine the most accurate methods for their particular problems. For useful parts that are not well represented by an ideal specimen, we must apply modifying factors as shown in the equation below:

$$S_e = k_a k_b k_c S'_e$$

Here k_a represents the *surface condition modification factor.* It is calculated using the ultimate tensile strength of the material, S_u, and factors a and b found in Table 6-2 of Shigley's Mechanical Engineering Design [Budynas and Nisbett, 2006]. Factors a and b are dependent on the type of surface finish (ground, machined, hot-rolled, etc.) and the system of measurement being used (SI or English). The equation for k_a is shown below:

$$k_a = aS_u^b$$

The *size modification factor*, k_b acounts for differences in part sizes compared to those used in experiements. For axial loading $k_b = 1$. For bending and torsion, k_b depends on the size of the part. Equations shown here are only accurate for diameters, d, measured in inches [Budynas and Nisbett, 2006].

$$k_b = \left(\frac{d}{0.3}\right)^{-0.107} = 0.879 \cdot d^{-0.107} \quad \text{for} \quad 0.11 \leq d \leq 2.0 \text{ in}$$

$$k_b = 0.91 d^{-0.0157} \qquad \text{for} \quad 2.0 < d < 10.0 \text{ in}$$

K_c is the *loading factor*. The endurance limit proves to be different for each kind of loading. We adjust for this by including a loading factor in our calculations.

$$k_c = 1.0 \quad \text{bending}$$

$$k_c = 0.85 \quad \text{axial}$$

$$k_c = 0.59 \quad \text{pure torsion}$$

Example of endurance limit analysis:

- Material: Alum 7075-T6, $S'_e = 23$ ksi. This is the endurance strength under ideal conditions. Ultimate tensile strength, used for some calculations below, $S_u = 83\,ksi$.
- Fatigue strength modification factors:
  - loading, bending with $k_c = 1$;
  - surface, ground, $a = 1.34$, $b = -0.08$, $k_a = aS_u^b = 0.94$;
  - size, diameter $d = 1$ in., $k_b = 0.879d^{-0.107} = 0.88$.
- Part stress: peak bending stress $\sigma_{max} = 9$ ksi, no concentrators.
- Synthesis: $S_e = k_a k_b k_c S'_e = 18$ ksi, thus, $F.O.S. = S_e/\sigma_{max} = 2$ for unlimited cycles.

Note: This example is not representative of fatigue analysis for any given part. Many more modification factors for calculating S_e exist for different operating conditions which should be considered. Other methods not shown here may be more useful depending on the problem. The endurance limit is an important value to calculate for parts experiencing extremely large numbers of cycles, but not for parts experiencing for few cycles. A more compelte list of methods and modification factors can be found in other sources such as Shigley's Mechanical Engineering Design[Budynas and Nisbett, 2006].

4.3.3 Fatigue Concepts for Design

With cyclic loading, peak stress matters, but is compared to an adjusted allowable stress. Get a rough idea of the factors that affect this allowable stress in your design domain and avoid designs that exacerbate such factors. When the qualitative design is set, estimate S and optimize the design, then check S with the final parameter values. Fatigue is harder to predict than static failure, so adjust the factor of safety of your design accordingly. In life-critical applications, always perform tests on a physical prototype before going to production. Remember that:

- Material matters
- Stress concentrations matter
- Other factors (e.g. surface, size, temperature, etc.) matter some

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