Carnegie Mellon
School of Computer Science

Deep Reinforcement Learning and Control

Policy gradients

CMU 10-403

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Used Materials

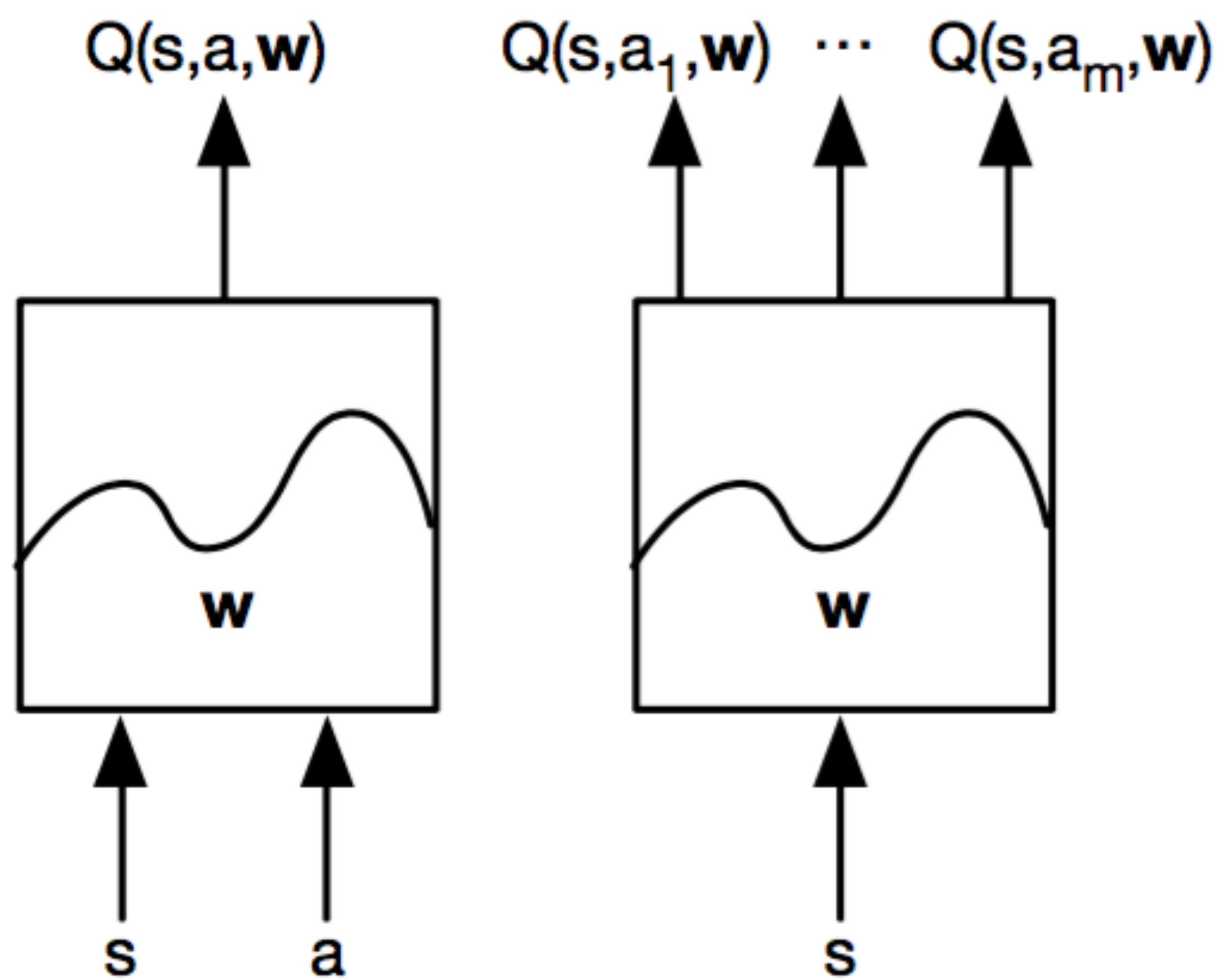
- **Disclaimer**: Much of the material and slides for this lecture were borrowed from Russ Salakhutdinov, Rich Sutton's class and David Silver's class on Reinforcement Learning.

Revision

Deep Q-Networks (DQNs)

- Represent action-state value function by Q-network with weights w

$$Q(s, a, \mathbf{w}) \approx Q^*(s, a)$$



Cost function

- Minimize mean-squared error between the true action-value function $q_\pi(S,A)$ and the approximate Q function:

$$J(\mathbf{w}) = \mathbb{E}_\pi\left[\left(q_\pi(S,A) - Q(S,A,\mathbf{w})\right)^2\right]$$

- We do not know the groundtruth value
- Minimize MSE loss by stochastic gradient descent

$$\mathscr{L} = \left(r + \gamma \max_{a'} Q(s,a',\mathbf{w}) - Q(s,a,\mathbf{w})\right)^2$$

wrong!

Cost function

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Q-Learning: Off-Policy TD Control

- One-step Q-learning:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha\Big[R_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t)\Big]$$

Initialize $Q(s, a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
    Initialize S
    Repeat (for each step of episode):
        Choose A from S using policy derived from Q (e.g., ε-greedy)
        Take action A, observe R, S'
        $Q(S, A) \leftarrow Q(S, A) + \alpha[R + \gamma \max_a Q(S', a) - Q(S, A)]$
        $S \leftarrow S'$;
    until S is terminal

Stability of training problems for DQN

- Minimize MSE loss by stochastic gradient descent

$$\mathscr{L} = \left(r + \gamma \max_{a'} Q(s', a', \mathbf{w}) - Q(s, a, \mathbf{w}) \right)^2$$

- Converges to Q_* using table lookup representation

- But **diverges** using neural networks due to:
  1. Correlations between samples
  2. Non-stationary targets

- Solutions:
  1. Experience buffer
  2. Targets stay fixed for many iterations

Learning a DQN supervised from a planner

- Minimize MSE loss by stochastic gradient descent

$$\mathscr{L} = \left(Q_{MCTS}(s,a) - Q(s,a,\mathbf{w})\right)^2$$

- Boils down to a supervised learning problem
- I use MCTS to play 800 games, I gather the Q estimates of states and actions in the MCTS trees and train a regressor.
- Any problems?
- Any solutions?
- DAGGER!

Learning a DQN supervised from a planner

- Minimize MSE loss by stochastic gradient descent

$$\mathscr{L} = \left(Q_{MCTS}(s, a) - Q(s, a, \mathbf{w})\right)^2$$

- Boils down to a supervised learning problem
- I use MCTS to play 800 games, I gather the Q estimates of states and actions in the MCTS trees and train a regressor. Then use it to find a policy
- Any problems?
- Any solutions?
- DAGGER!
- Also: training a classifier directly worked best!

Policy-Based Reinforcement Learning

- So far we approximated the value or action-value function using parameters θ (e.g. neural networks)

$$Q_\theta(s, a) \approx Q^\pi(s, a)$$

- A policy was generated directly from the value function e.g. using ε-greedy

- In this lecture we will directly parameterize the policy

$$\pi_\theta(s, a) = \mathbb{P}\left[a \mid s, \theta\right]$$

- We will not use any models, and we will learn from experience, not imitation

Policy-Based Reinforcement Learning

- So far we approximated the value or action-value function using parameters θ (e.g. neural networks)

$$V_\theta(s) \approx V^\pi(s)$$

$$Q_\theta(s,a) \approx Q^\pi(s,a)$$

- A policy … using ε-greedy

- In this le…

Sometimes I will also use the notation:

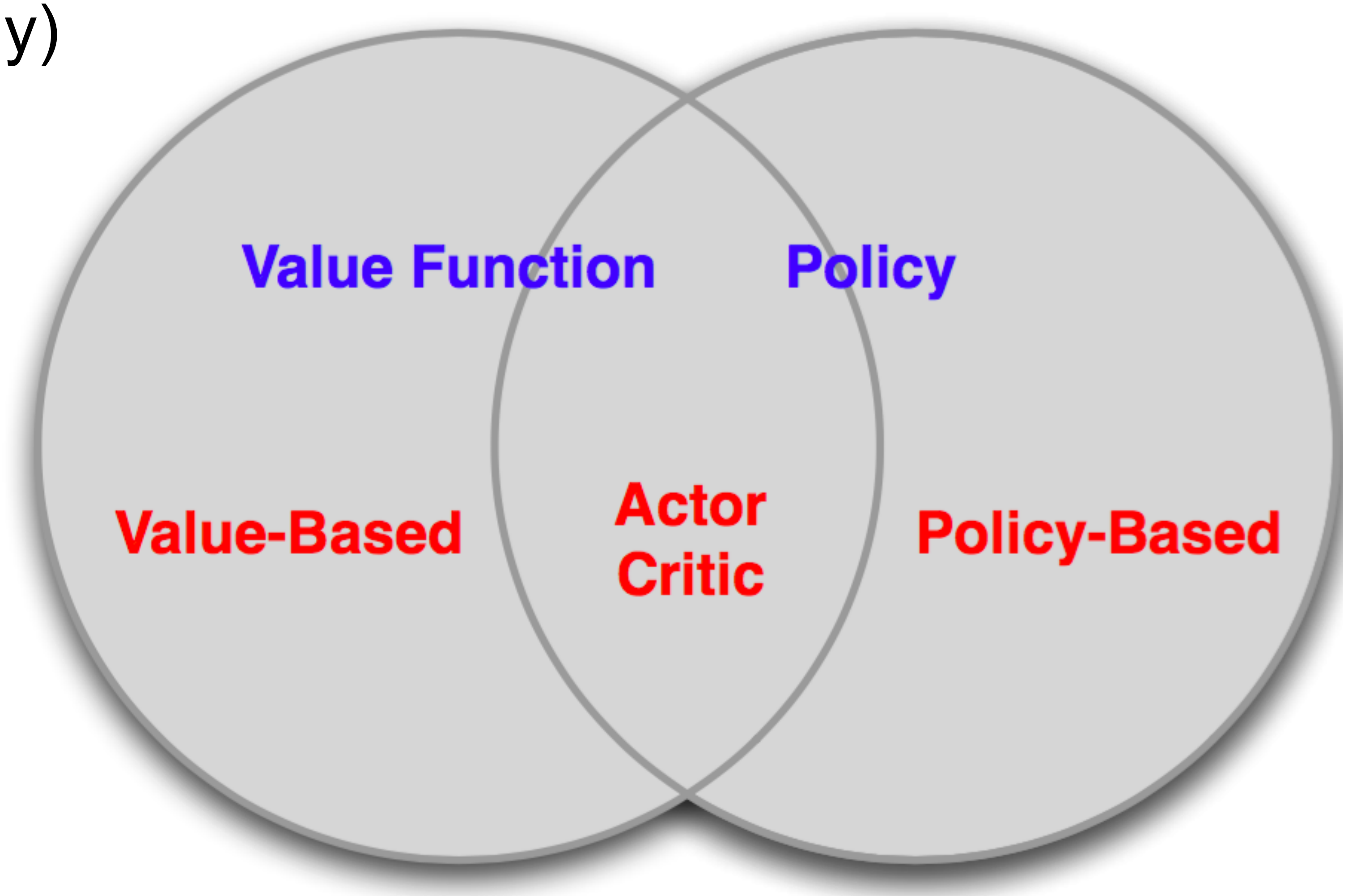
$$\pi(A_t|S_t,\boldsymbol{\theta})$$

$$\pi_\theta(s,a) = \mathbb{P}\left[a \mid s,\theta\right]$$

- We will focus again on model-free reinforcement learning

Value-Based and Policy-Based RL

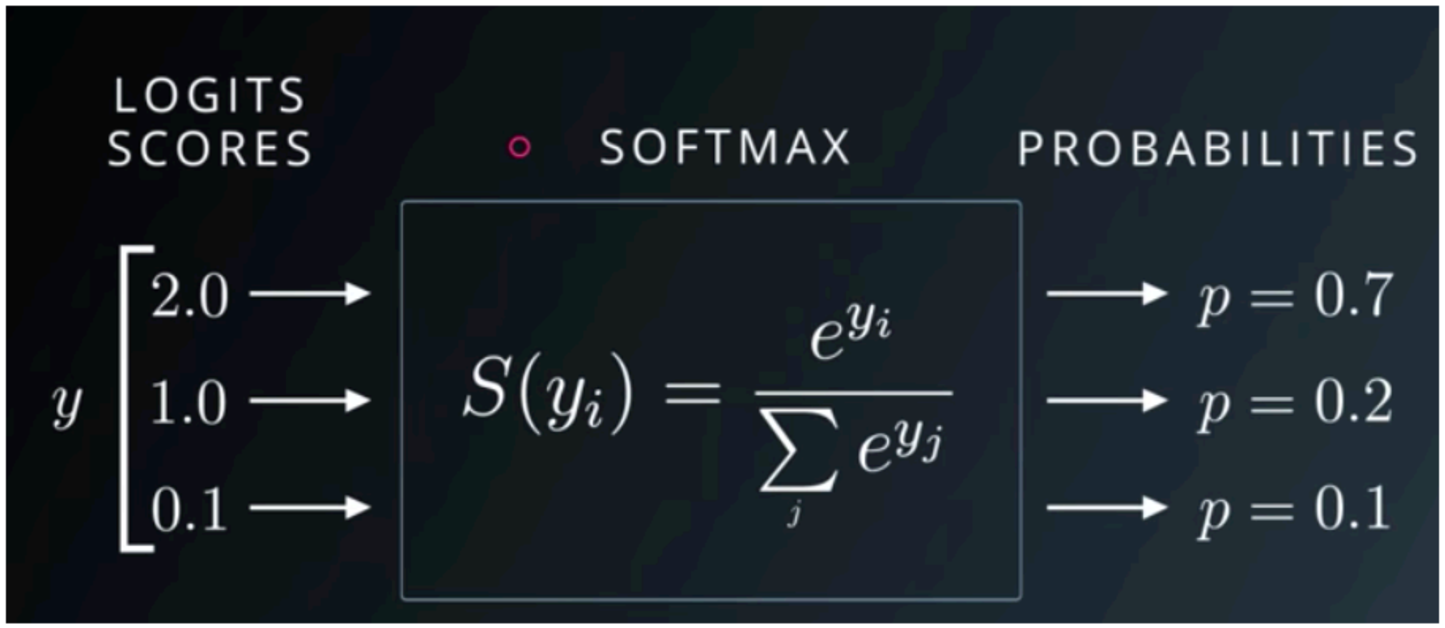
- Value Based
  - Learned Value Function
  - Implicit policy (e.g. ε-greedy)
- Policy Based
  - No Value Function
  - Learned Policy
- Actor-Critic
  - Learned Value Function
  - Learned Policy



Advantages of Policy-Based RL

- Advantages
  - Effective in high-dimensional or continuous action spaces
  - Can learn stochastic policies
  - We will look into the benefits of stochastic policies in a future lecture

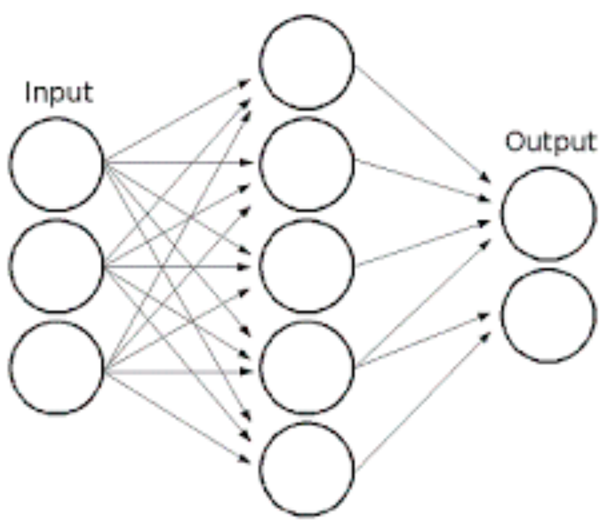
Policy function approximators



$$\pi(a|s,\boldsymbol{\theta}) \doteq \frac{e^{h(s,a,\boldsymbol{\theta})}}{\sum_b e^{h(s,b,\boldsymbol{\theta})}}$$

discrete actions

s



go left

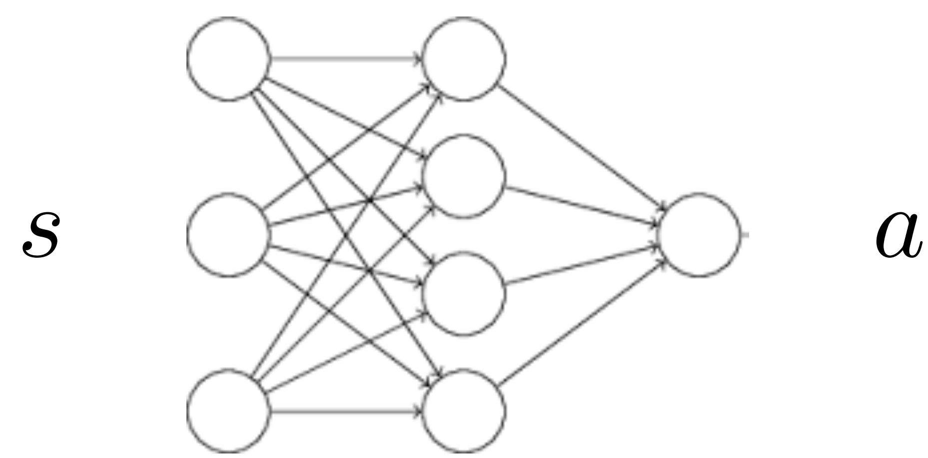
go right

Output is a distribution over a discrete set of actions

With continuous policy parameterization the action probabilities change smoothly as a function of the learned parameter, whereas in epsilon-greedy selection the action probabilities may change dramatically for an arbitrarily small change in the estimated action values, if that change results in a different action having the maximal value.

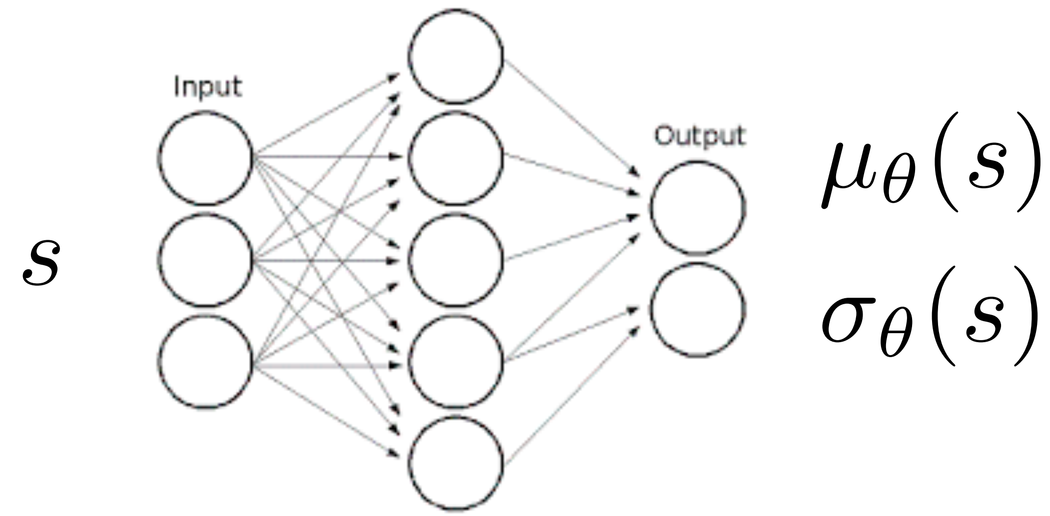
Policy function approximators

deterministic continuous policy



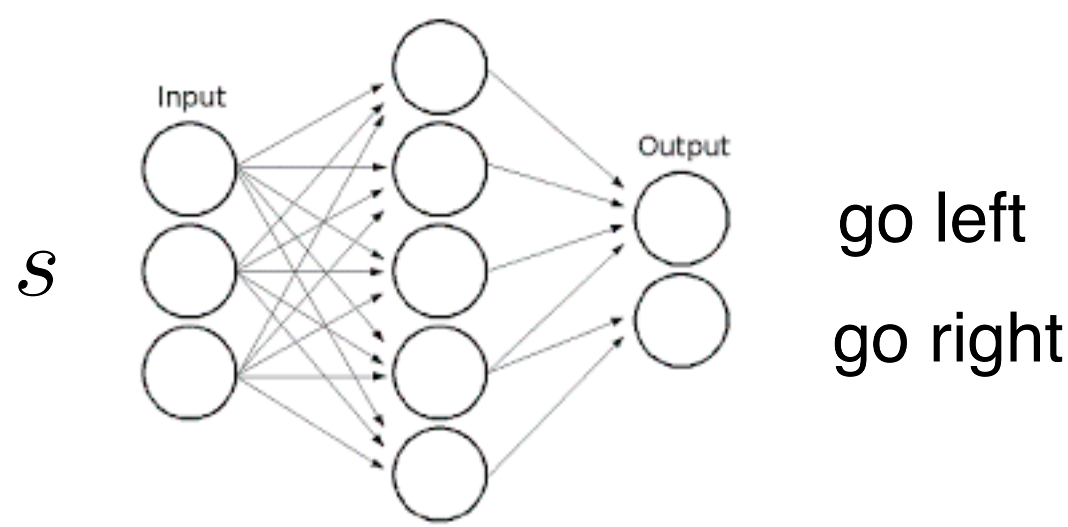
$$a = \pi_\theta(s)$$

stochastic continuous policy



$$a \sim \mathcal{N}(\mu_\theta(s), \sigma^2_\theta(s))$$

discrete actions



Output is a distribution over a discrete set of actions

Policy Objective Functions

- Goal: given policy $\pi_\theta(s,a)$ with parameters θ, find best θ
- But how do we measure the quality of a policy π_θ?
- In episodic environments we can use the start value

$$J_1(\theta) = V^{\pi_\theta}(s_1) = \mathbb{E}_{\pi_\theta}[v_1]$$

- In continuing environments we can use the average value

$$J_{avV}(\theta) = \sum_s d^{\pi_\theta}(s) V^{\pi_\theta}(s)$$

- Or the average reward per time-step

$$J_{avR}(\theta) = \sum_s d^{\pi_\theta}(s) \sum_a \pi_\theta(s,a)\mathcal{R}_s^a$$

where $d^{\pi_\theta}(s)$ is stationary distribution of Markov chain for π_θ

Policy Objective Functions

- Goal: given policy $\pi_\theta(s,a)$ with parameters θ, find best θ
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- In continuing environments we can use the average value

$$J_{avV}(\theta) = \sum_s d^{\pi_\theta}(s) V^{\pi_\theta}(s)$$

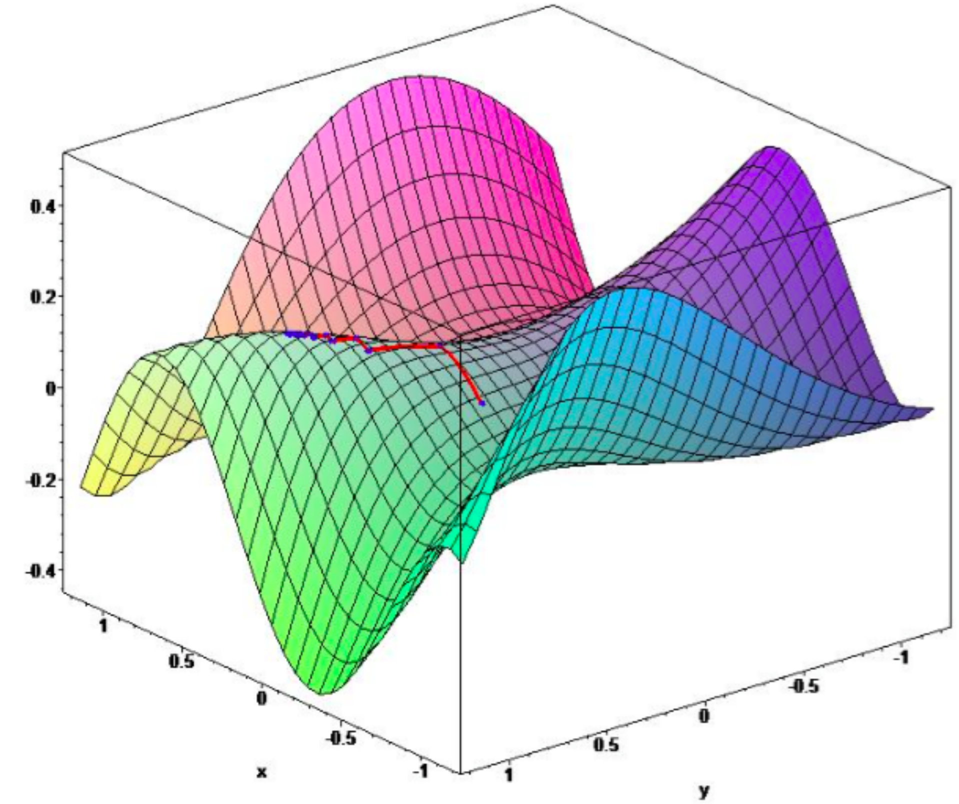
- In the episodic case, $d^{\pi_\theta}(s)$ is defined to be
  - the expected number of time steps t on which $S_t = s$
  - in a randomly generated episode starting in s_0 and
  - following π and the dynamics of the MDP.

Remember: Episode of experience under policy π: $S_1, A_1, R_2, \ldots, S_k \sim \pi$

Policy Optimization

- Policy based reinforcement learning is an optimization problem
  - Find θ that maximizes $J(\theta)$
- Some approaches do not use gradient
  - Hill climbing
  - Genetic algorithms
- Greater efficiency often possible using gradient
- We focus on gradient descent, many extensions possible
- And on methods that exploit sequential structure

Policy Gradient



- Let J(θ) be any policy objective function
- Policy gradient algorithms search for a local maximum in J(θ) by ascending the gradient of the policy, w.r.t. parameters θ

$$\Delta\theta = \alpha \nabla_\theta J(\theta)$$

α is a step-size parameter (learning rate)

is the policy gradient

$$\nabla_\theta J(\theta) = \begin{pmatrix} \frac{\partial J(\theta)}{\partial \theta_1} \\ \vdots \\ \frac{\partial J(\theta)}{\partial \theta_n} \end{pmatrix}$$

Computing Gradients By Finite Differences

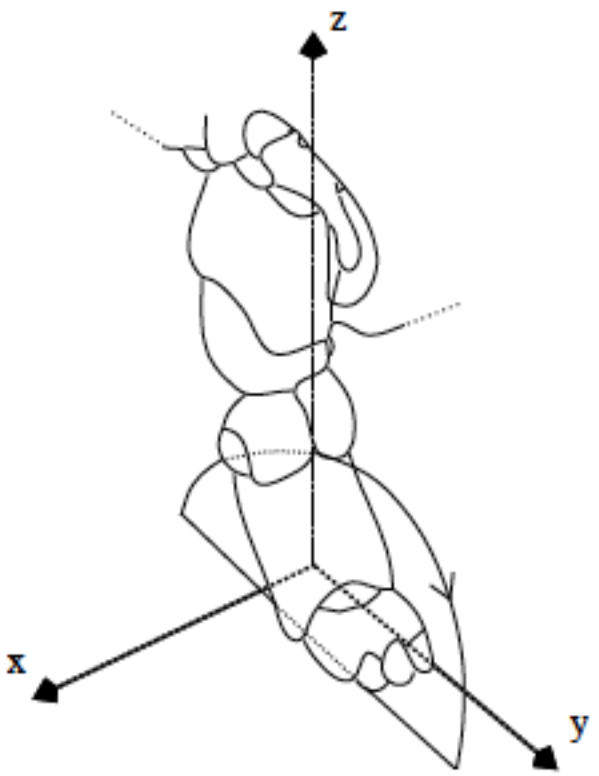
- To evaluate policy gradient of $\pi_\theta(s, a)$
- For each dimension k in [1, n]
  - Estimate k^{th} partial derivative of objective function w.r.t. θ
  - By perturbing θ by small amount ε in k^{th} dimension

$$\frac{\partial J(\theta)}{\partial \theta_k} \approx \frac{J(\theta + \epsilon u_k) - J(\theta)}{\epsilon}$$

where u_k is a unit vector with 1 in k^{th} component, 0 elsewhere

- Uses n evaluations to compute policy gradient in n dimensions
- Simple, noisy, inefficient - but sometimes effective
- Works for arbitrary policies, even if policy is not differentiable

Learning an AIBO running policy



- Goal: learn a fast AIBO walk (useful for Robocup)
- AIBO walk policy is controlled by 12 numbers (elliptical loci)
- Adapt these parameters by finite difference policy gradient
- Evaluate performance of policy by field traversal time

Learning an AIBO running policy

$\pi \leftarrow InitialPolicy$
while !done **do**
  $\{R_1, R_2, \ldots, R_t\} = t$ random perturbations of π
  evaluate($\{R_1, R_2, \ldots, R_t\}$)
  for $n = 1$ to N **do**
    $Avg_{+\epsilon,n} \leftarrow$ average score for all R_i that have a positive perturbation in dimension n
    $Avg_{+0,n} \leftarrow$ average score for all R_i that have a zero perturbation in dimension n
    $Avg_{-\epsilon,n} \leftarrow$ average score for all R_i that have a negative perturbation in dimension n
    if $Avg_{+0,n} > Avg_{+\epsilon,n}$ and $Avg_{+0,n} > Avg_{-\epsilon,n}$ **then**
      $A_n \leftarrow 0$
    else
      $A_n \leftarrow Avg_{+\epsilon,n} - Avg_{-\epsilon,n}$
    end if
  end for
  $A \leftarrow \frac{A}{|A|} * \eta$
  $\pi \leftarrow \pi + A$
end while

Policy Gradient Reinforcement Learning for Fast Quadrupedal Locomotion, Kohl and Stone, 2004

Learning an AIBO running policy

Initial

Training

Final

Policy Gradient: Score Function

- We now compute the policy gradient analytically
- Assume
  - policy π_θ is differentiable whenever it is non-zero
  - we know the gradient $\nabla_\theta \pi_\theta(s, a)$
- Likelihood ratios exploit the following identity

$$\begin{aligned}\nabla_\theta \pi_\theta(s, a) &= \pi_\theta(s, a) \frac{\nabla_\theta \pi_\theta(s, a)}{\pi_\theta(s, a)} \\ &= \pi_\theta(s, a) \nabla_\theta \log \pi_\theta(s, a)\end{aligned}$$

- The score function is $\nabla_\theta \log \pi_\theta(s, a)$

Softmax Policy: Discrete Actions

- We will use a softmax policy as a running example
- Weight actions using linear combination of features $\phi(s, a)^\top \theta$
- Probability of action is proportional to exponentiated weight

$$\pi_\theta(s, a) \propto e^{\phi(s,a)^\top \theta}$$

Nonlinear extension: replace $\phi(s, a)$ with a deep neural network with trainable weights w

Think a neural network with a softmax output probabilities

Softmax Policy: Discrete Actions

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Think a neural network with a softmax output probabilities

- The score function is

$$\nabla_\theta \log \pi_\theta(s, a) = \phi(s, a) - \mathbb{E}_{\pi_\theta}[\phi(s, \cdot)]$$

Gaussian Policy: Continuous Actions

- In continuous action spaces, a Gaussian policy is natural

- Mean is a linear combination of state features

$$\mu(s) = \phi(s)^\top \theta$$

Nonlinear extension: replace $\phi(s)$ with a deep neural network with trainable weights w

$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- Variance may be fixed σ_2, or can also parameterized

- Policy is Gaussian $a \sim \mathcal{N}(\mu(s), \sigma^2)$

- The score function is

$$\nabla_\theta \log \pi_\theta(s, a) = \frac{(a - \mu(s))\phi(s)}{\sigma^2}$$

One-step MDP

- Consider a simple class of one-step MDPs
  - Starting in state $s \sim d(s)$
  - Terminating after one time-step with reward $r = \mathcal{R}_{s,a}$
- First, let's look at the objective:

$$J(\theta) = \mathbb{E}_{\pi_\theta}[r]$$
$$= \sum_{s \in \mathcal{S}} d(s) \sum_{a \in \mathcal{A}} \pi_\theta(s,a)\mathcal{R}_{s,a}$$

Intuition: Under MDP:

$$\mathbb{E}_{\pi_\theta}[r] = \sum_{s \in \mathcal{S}} \sum_{a \in \mathcal{A}} P_\theta(s,a)\mathcal{R}_{s,a} = \sum_{s \in \mathcal{S}} \sum_{a \in \mathcal{A}} P(s)\pi_\theta(a|s)\mathcal{R}_{s,a}$$
$$= \sum_{s \in \mathcal{S}} P(s) \sum_{a \in \mathcal{A}} \pi_\theta(a|s)\mathcal{R}_{s,a}$$

One-step MDP

- Consider a simple class of one-step MDPs
  - Starting in state $s \sim d(s)$
  - Terminating after one time-step with reward $r = \mathcal{R}_{s,a}$
- Use likelihood ratios to compute the policy gradient

$$
\begin{aligned}
J(\theta) &= \mathbb{E}_{\pi_\theta}[r] \\
&= \sum_{s \in \mathcal{S}} d(s) \sum_{a \in \mathcal{A}} \pi_\theta(s,a) \mathcal{R}_{s,a}
\end{aligned}
$$

$$
\begin{aligned}
\nabla_\theta J(\theta) &= \sum_{s \in \mathcal{S}} d(s) \sum_{a \in \mathcal{A}} \pi_\theta(s,a) \nabla_\theta \log \pi_\theta(s,a) \mathcal{R}_{s,a} \\
&= \mathbb{E}_{\pi_\theta}[\nabla_\theta \log \pi_\theta(s,a) r]
\end{aligned}
$$