Carnegie Mellon
School of Computer Science

Deep Reinforcement Learning and Control

Policy gradients

CMU 10-403

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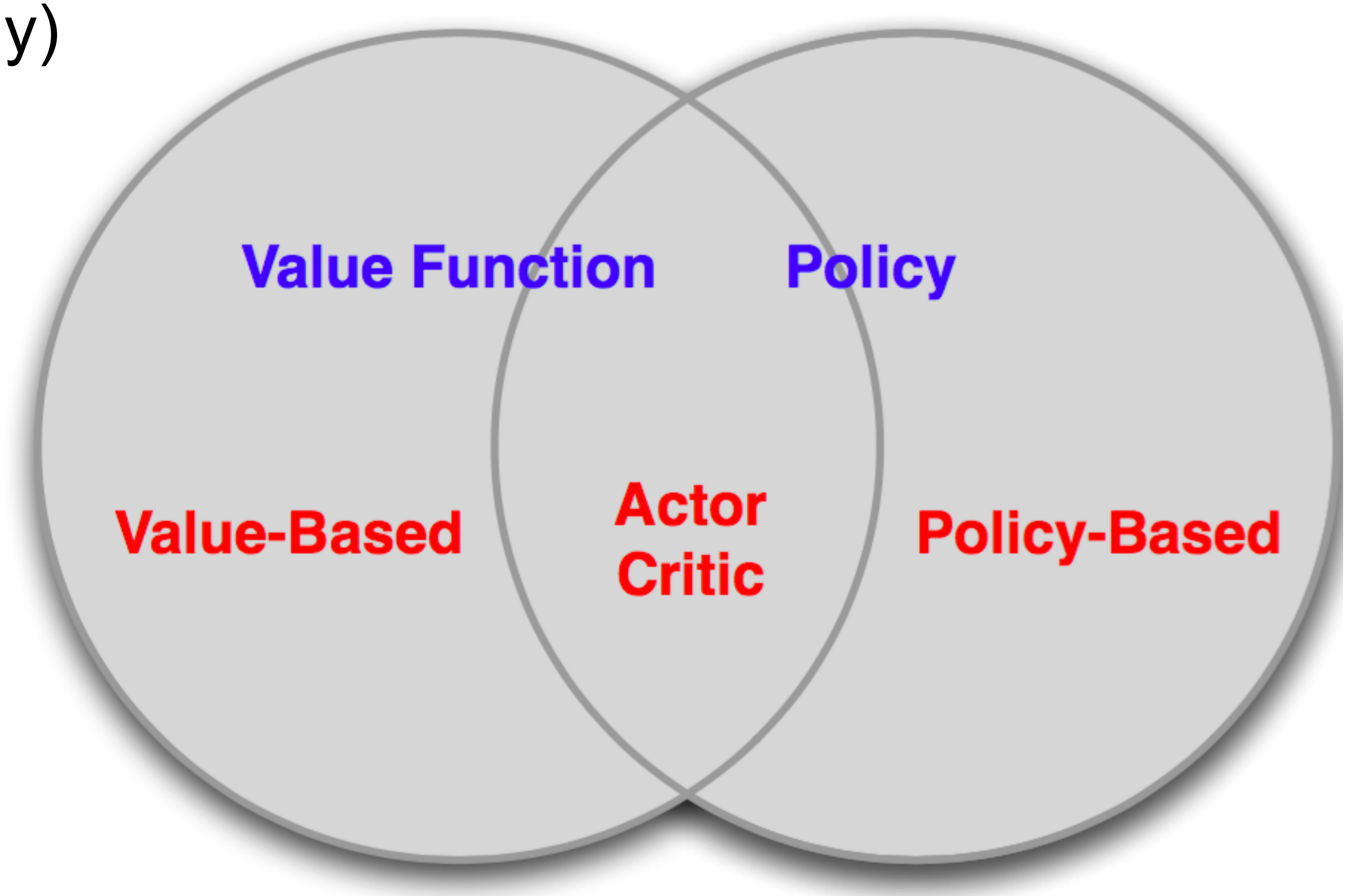
Revision

Value-Based and Policy-Based RL

- Value Based
  - Learned Value Function
  - Implicit policy (e.g. ε-greedy)
- Policy Based
  - No Value Function
  - Learned Policy
- Actor-Critic
  - Learned Value Function
  - Learned Policy

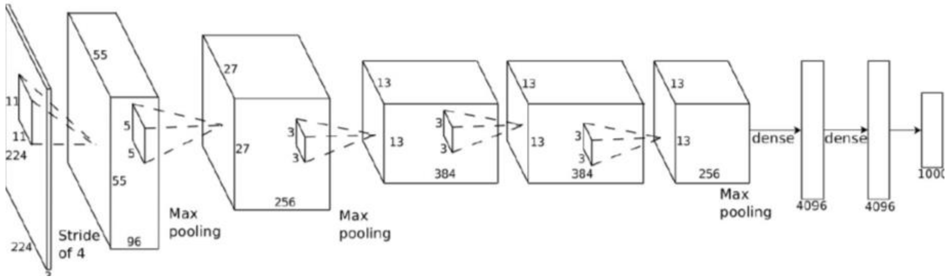


Advantages of Policy-Based RL

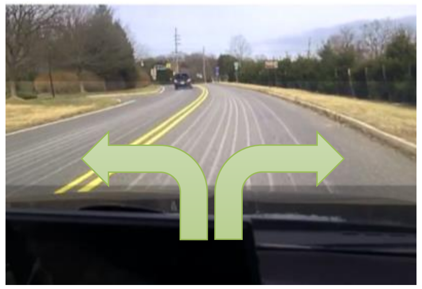
- Advantages
  - Effective in high-dimensional or continuous action spaces
  - Can learn stochastic policies

Policy function approximators

$\mathbf{o}_t$



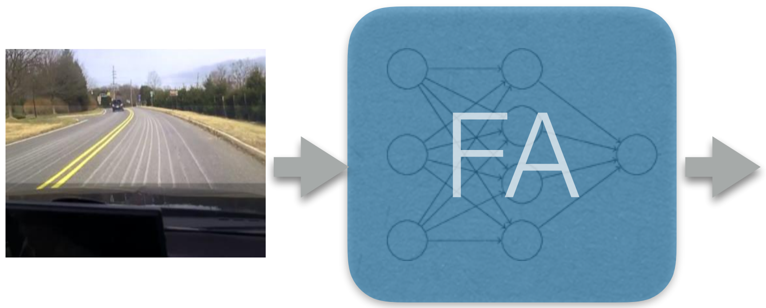
$\pi_\theta(\mathbf{u}_t|\mathbf{o}_t)$



$\mathbf{u}_t$

Policy function approximators

deterministic continuous policy

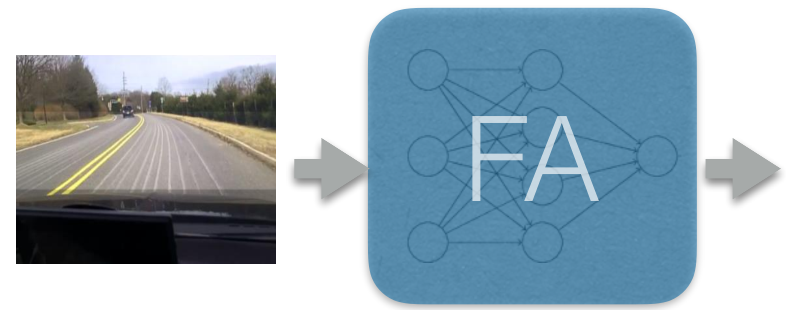


a

$$a = \pi_\theta(s)$$

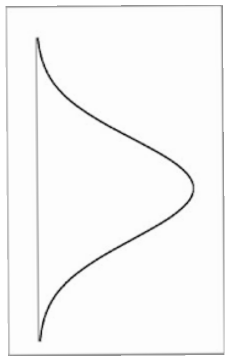
e.g. outputs a steering angle directly

stochastic continuous policy



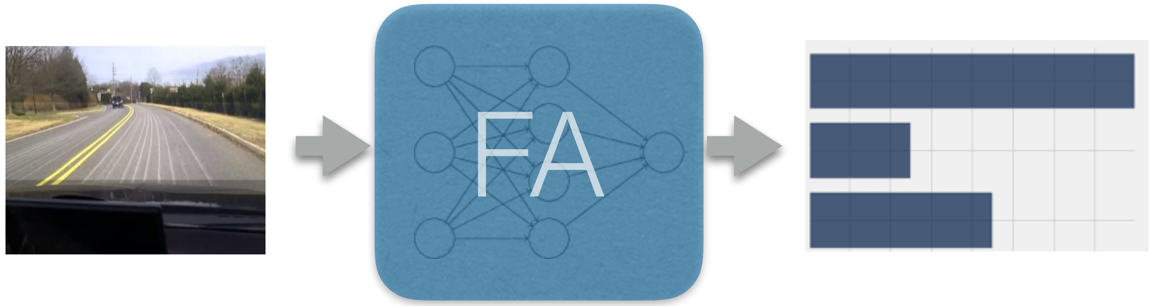
$\mu_\theta(s)$

$\sigma_\theta(s)$



$$a \sim \mathcal{N}(\mu_\theta(s), \sigma_\theta^2(s))$$

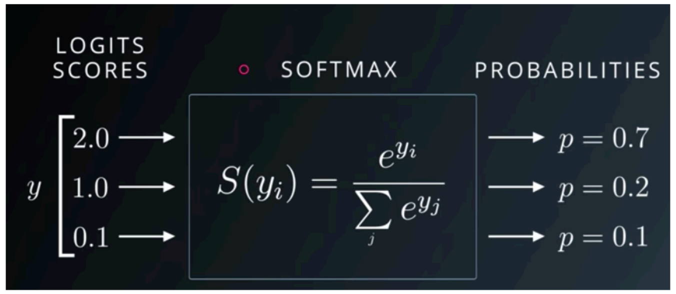
(stochastic) discrete actions



go left

go right

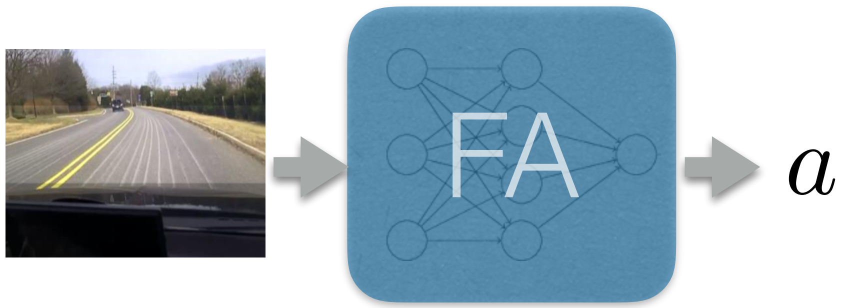
press brake



Outputs a distribution over a discrete set of actions

Policy function approximators - this lecture

deterministic continuous policy



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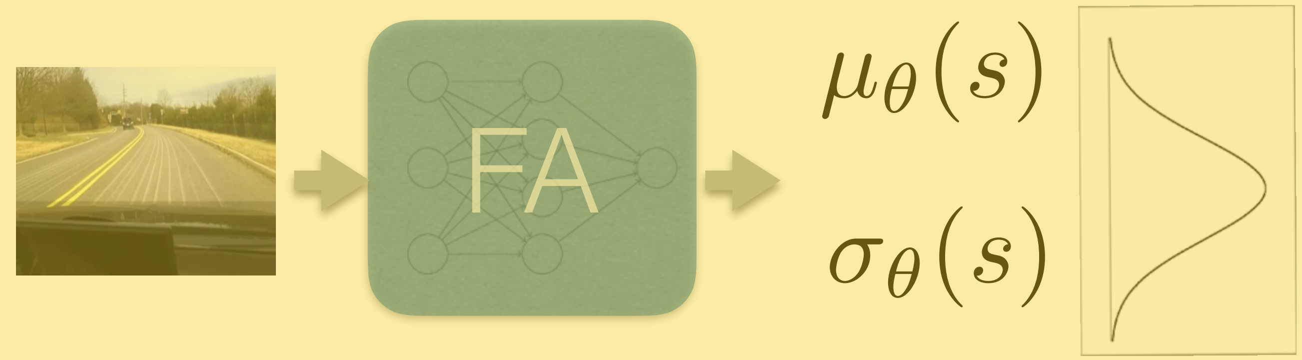

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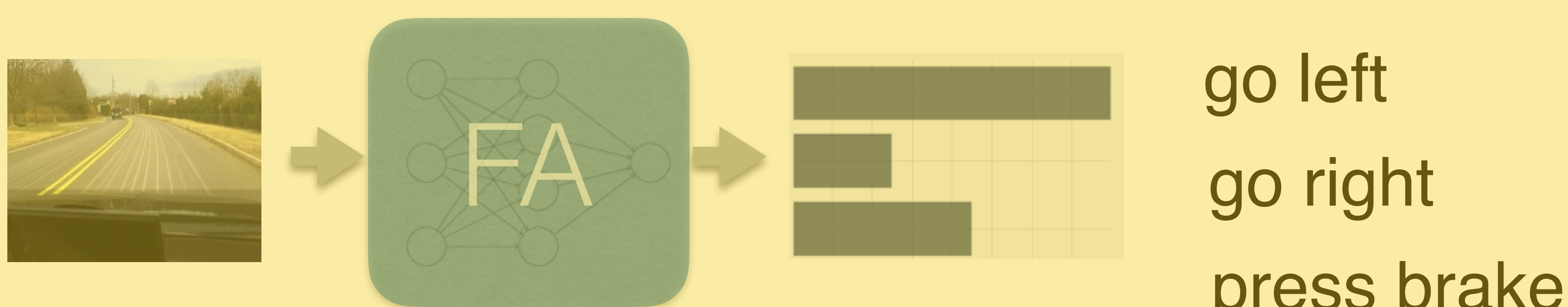
(stochastic) discrete actions



Outputs a distribution over a discrete set of actions

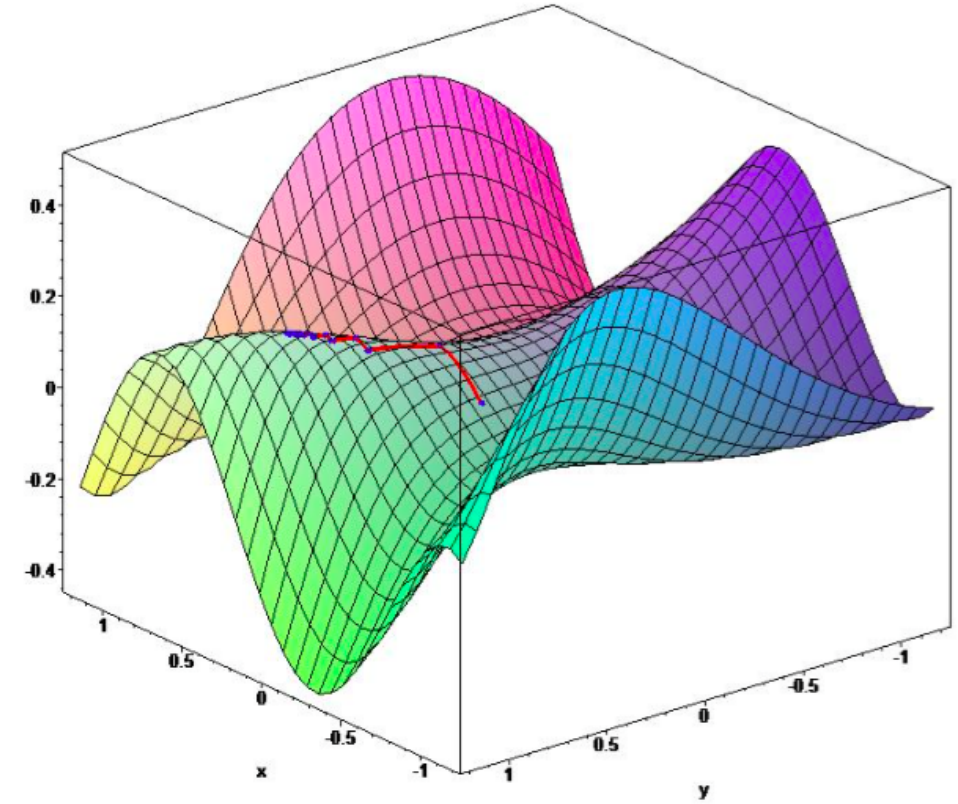
Policy Optimization

- Let U(θ) be any policy **objective function**
- Policy based reinforcement learning is an optimization problem
  - Find θ that maximizes U(θ)
- Some approaches do not use gradient
  - Hill climbing
  - Genetic algorithms
- Greater efficiency often possible using gradient

Policy Gradient

- Let U(θ) be any policy **objective function**

- Policy gradient algorithms search for a local maximum in U(θ) by ascending the gradient of the policy, w.r.t. parameters θ



$$\theta_{new} = \theta_{old} + \Delta\theta$$

$$\Delta\theta = \alpha \nabla_\theta U(\theta)$$

α is a step-size parameter (learning rate)

is the policy gradient

$$\nabla_\theta U(\theta) = \begin{pmatrix} \frac{\partial U(\theta)}{\partial \theta_1} \\ \vdots \\ \frac{\partial U(\theta)}{\partial \theta_n} \end{pmatrix}$$

Policy gradient: the gradient of the policy objective w.r.t. the parameters of the policy

Computing Gradients By Finite Differences

- Numerically approximating the policy gradient of $\pi_\theta(s, a)$
- For each dimension k in [1, n]
  - Estimate k^{th} partial derivative of objective function w.r.t. θ
  - By perturbing θ by small amount ε in k^{th} dimension

$$\frac{\partial \mathrm{U}(\theta)}{\partial \theta_k} \approx \frac{\mathrm{U}(\theta + \epsilon u_k) - \mathrm{U}(\theta)}{\epsilon}$$

where u_k is a unit vector with 1 in k^{th} component, 0 elsewhere

- Uses n evaluations to compute policy gradient in n dimensions
- Simple, noisy, inefficient - but sometimes effective
- Works for arbitrary policies, even if policy is not differentiable

Learning an AIBO running policy

$\pi \leftarrow InitialPolicy$
while !done **do**
  $\{R_1, R_2, \ldots, R_t\} = t$ random perturbations of π
  evaluate($\{R_1, R_2, \ldots, R_t\}$)
  for k $= 1$ to N **do**
    $Avg_{+\epsilon,k} \leftarrow$ average score for all R_i that have a positive perturbation in dimension k
    $Avg_{+0,k} \leftarrow$ average score for all R_i that have a zero perturbation in dimension k
    $Avg_{-\epsilon,k} \leftarrow$ average score for all R_i that have a negative perturbation in dimension k
    if $Avg_{+0,k} > Avg_{+\epsilon,k}$ and $Avg_{+0,k} > Avg_{-\epsilon,k}$ **then**
      $A_k \leftarrow 0$
    else
      $A_k \leftarrow Avg_{+\epsilon,k} - Avg_{-\epsilon,k}$
    end if
  end for
  $A \leftarrow \frac{A}{|A|} * \eta$
  $\pi \leftarrow \pi + A$
end while

Policy Gradient Reinforcement Learning for Fast Quadrupedal Locomotion, Kohl and Stone, 2004

Policy objective

Trajectory τ is a state action sequence $s_0, a_0, s_1, a_1, \ldots s_H, a_H$

Trajectory reward: $R(\tau) = \sum_{t=0}^{H} R(s_t, a_t)$

A reasonable policy objective then is $U(\theta) = \mathbb{E}_{\tau \sim P(\tau;\theta)} R(\tau)$

$$\max_{\theta} . \quad U(\theta) = \mathbb{E}_{\tau \sim P(\tau;\theta)}[R(\tau)] = \sum_{\tau} P(\tau;\theta) R(\tau)$$

Probability of a trajectory: $P(\tau;\theta) = \prod_{t=0}^{H} \underbrace{P(s_{t+1} \mid s_t, a_t)}_{\text{dynamics}} \cdot \underbrace{\pi_\theta(a_t \mid s_t)}_{\text{policy}}$

Our problem is to compute $\nabla_\theta U(\theta) = \nabla_\theta \mathbb{E}_{\tau \sim P(\tau;\theta)}[R(\tau)]$

This lecture

Computing derivatives of expectations w.r.t. variables that parameterize the distribution, not the quantity inside the expectation

$$\max_{\theta} . \quad \mathbb{E}_{x \sim P(x;\theta)} f(x)$$

Assumptions:

- P is a probability density function that is continuous and differentiable
- P is easy to sample from

$$\max_{\theta} . \ \mathbb{E}_{\tau \sim P_{\theta}(\tau)} \left[R(\tau) \right]$$

Derivatives of expectations

$$\nabla_\theta \mathbb{E}_x f(x) = \nabla_\theta \mathbb{E}_{x \sim P_\theta(x)} \left[f(x) \right]$$

Derivatives of expectations

$$
\begin{aligned}
\nabla_\theta \mathbb{E}_x f(x) &= \nabla_\theta \mathbb{E}_{x \sim P_\theta(x)} \left[f(x) \right] \\
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Why?

Derivatives of expectations

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What is the problem here?

Derivatives of expectations

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What have we achieved?

Derivatives of expectations

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From the law of large numbers, I can obtain an unbiased estimator for the gradient by sampling!

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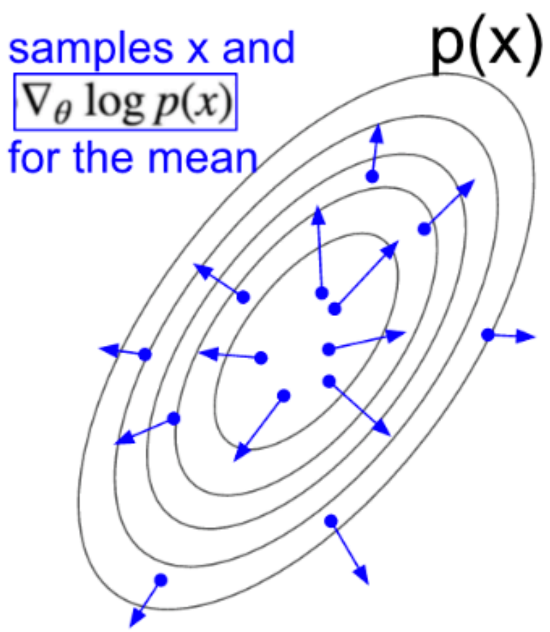
For Gaussian p(x)



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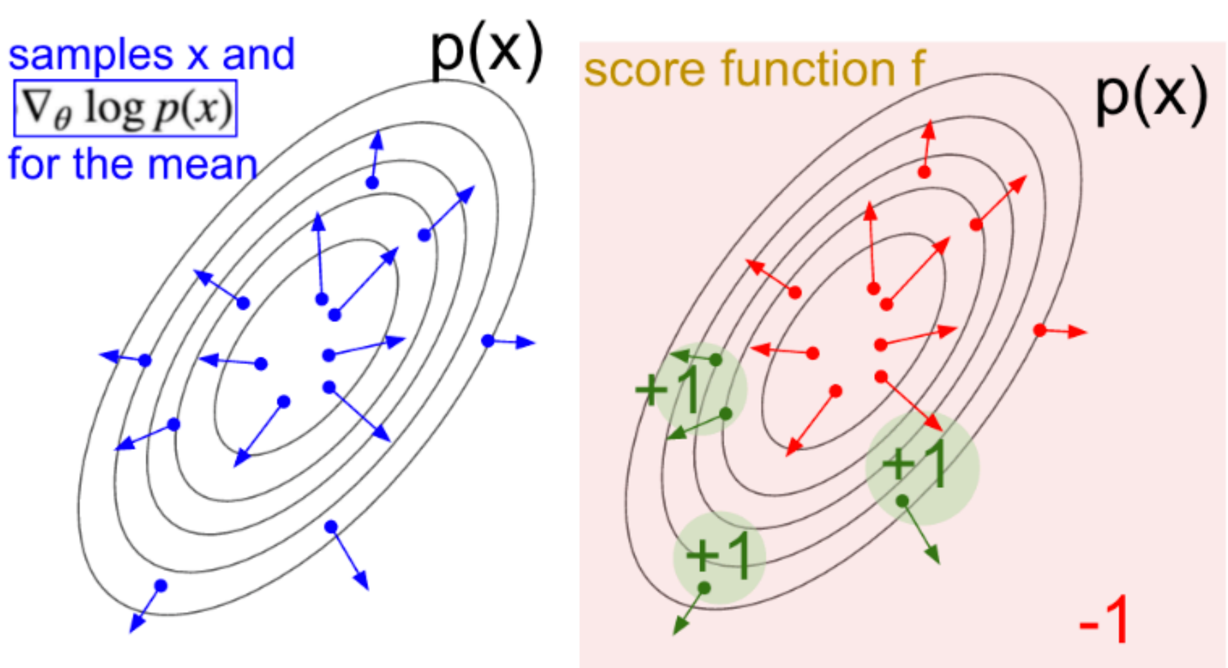

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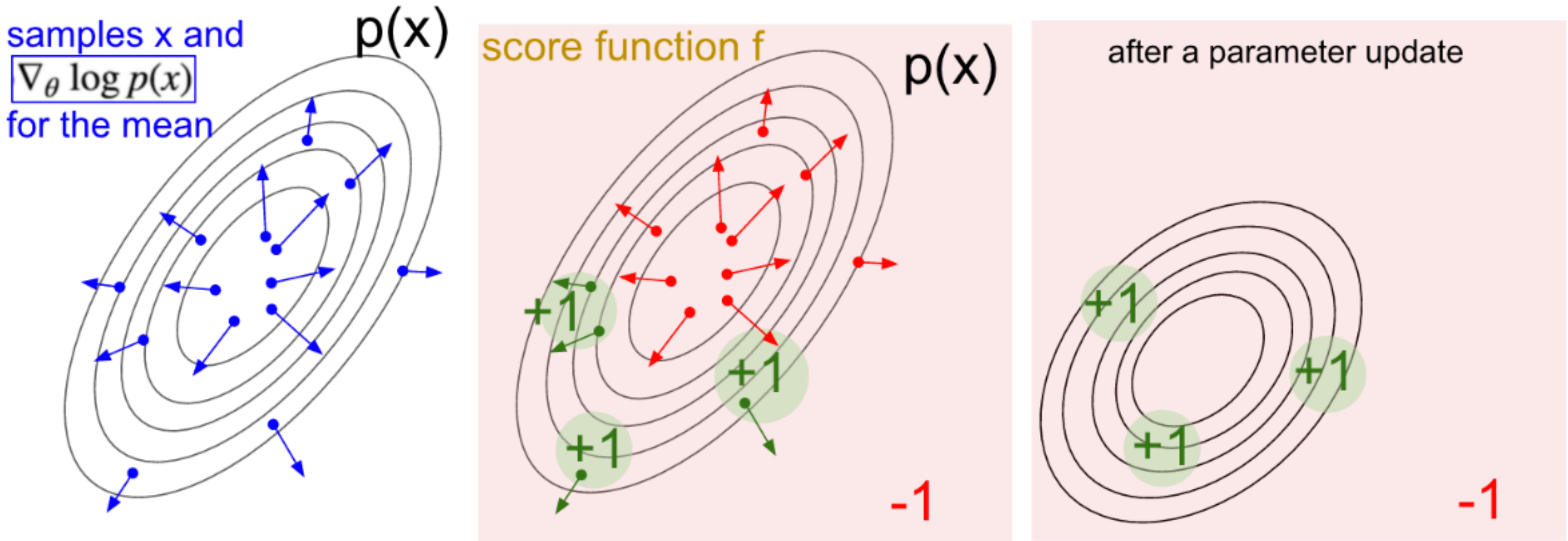

Derivatives of the policy objective

$$\max_{\theta} . \; U(\theta) = \mathbb{E}_{\tau \sim P_{\theta}(\tau)} \left[R(\tau) \right]$$

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&= \mathbb{E}_{\tau \sim P_\theta(\tau)} \left[\nabla_\theta \log P_\theta(\tau) R(\tau) \right]
\end{aligned}$$

Approximate the gradient with empirical estimate from N sampled trajectories:

$$\nabla_\theta U(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_\theta \log P_\theta(\tau^{(i)}) R(\tau^{(i)})$$

From trajectories to actions

$$\nabla_\theta \log P(\tau^{(i)};\theta) = \nabla_\theta \log \left[\prod_{t=0}^{T} \underbrace{P(s_{t+1}^{(i)} \mid s_t^{(i)}, a_t^{(i)})}_{\text{dynamics}} \cdot \underbrace{\pi_\theta(a_t^{(i)} \mid s_t^{(i)})}_{\text{policy}} \right]$$

$$= \nabla_\theta \left[\sum_{t=0}^{T} \underbrace{\log P(s_{t+1}^{(i)} \mid s_t^{(i)}, a_t^{(i)})}_{\text{dynamics}} + \underbrace{\log \pi_\theta(a_t^{(i)} \mid s_t^{(i)})}_{\text{policy}} \right]$$

$$= \nabla_\theta \left[\sum_{t=0}^{T} \underbrace{\log \pi_\theta(a_t^{(i)} \mid s_t^{(i)})}_{\text{policy}} \right]$$

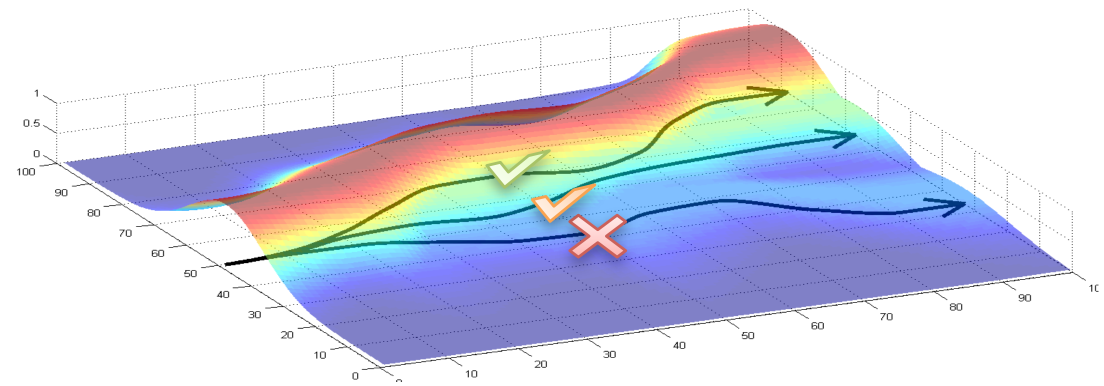
$$= \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t^{(i)} \mid s_t^{(i)})$$

$$\nabla_\theta U(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \nabla_\theta \log P_\theta(\tau^{(i)}) R(\tau^{(i)}) \quad \Longrightarrow \quad \nabla_\theta U(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\alpha_t^{(i)} \mid s_t^{(i)}) R(\tau^{(i)})$$

Intuition

$$\nabla_\theta U(\theta) \approx \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\alpha_t^{(i)} \mid s_t^{(i)}) R(\tau^{(i)})$$

- Gradient tries to:
  - Increase probability of paths with positive R
  - Decrease probability of paths with negative R



! Likelihood ratio changes probabilities of experienced paths, does not try to change the paths (<-> Path Derivative)

Likelihood ratio gradient estimator

$$\max_{\theta} . \ U(\theta) = \mathbb{E}_{\tau \sim P_\theta(\tau)} \left[R(\tau) \right]$$

$$\begin{aligned} \nabla_\theta U(\theta) &= \nabla_\theta \mathbb{E}_{\tau \sim P_\theta(\tau)} \left[R(\tau) \right] \\ &= \mathbb{E}_{\tau \sim P_\theta(\tau)} \left[\nabla_\theta \log P_\theta(\tau) R(\tau) \right] \end{aligned}$$

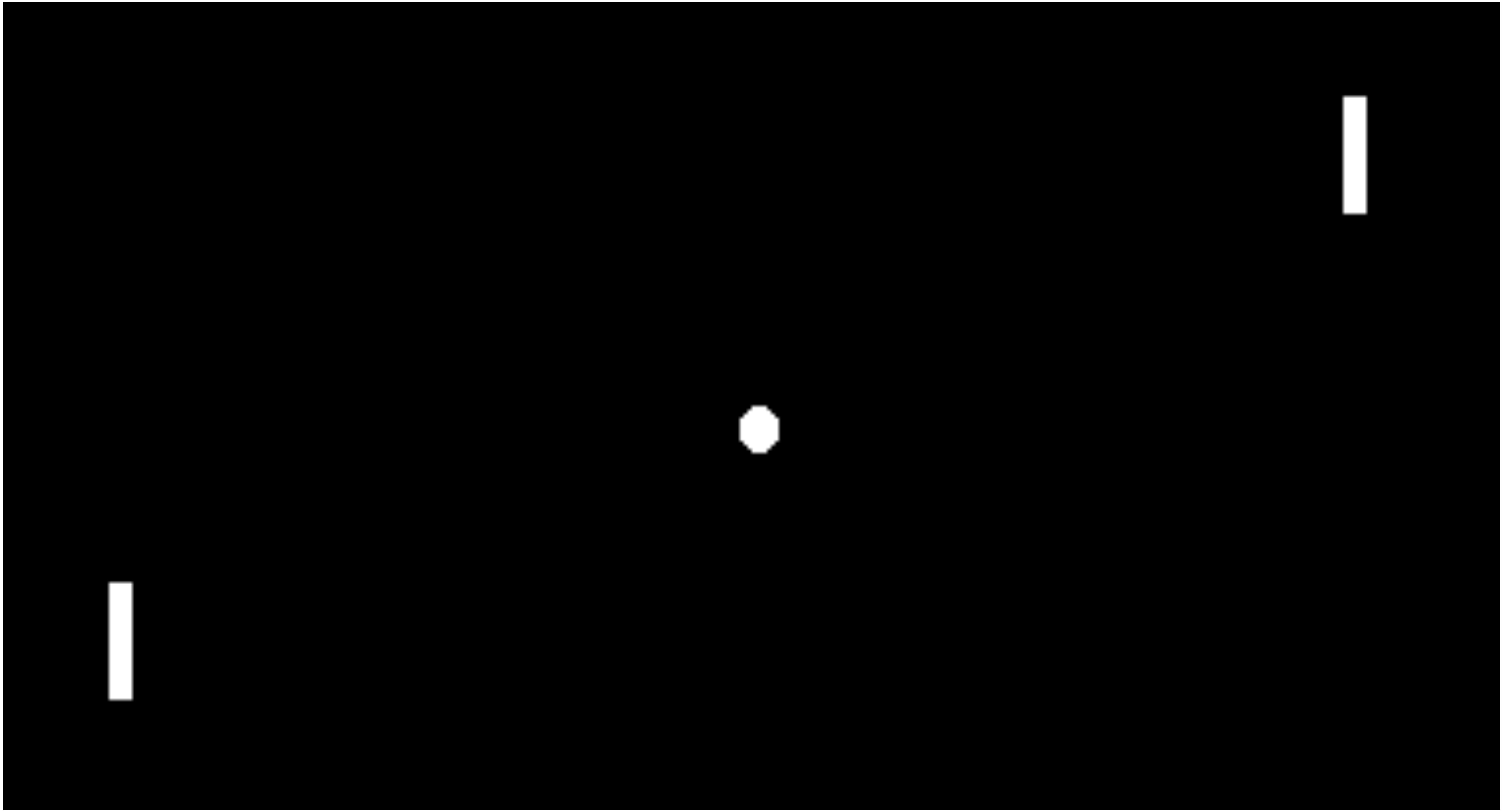
An unbiased estimator of this gradient:

$$\hat{g} = \frac{1}{N} \sum_{i=1}^{N} \nabla_\theta \log P_\theta(\tau^{(i)}) R(\tau^{(i)}) = \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_\theta \log \pi_\theta(\alpha_t^{(i)} \mid s_t^{(i)}) R(\tau^{(i)})$$

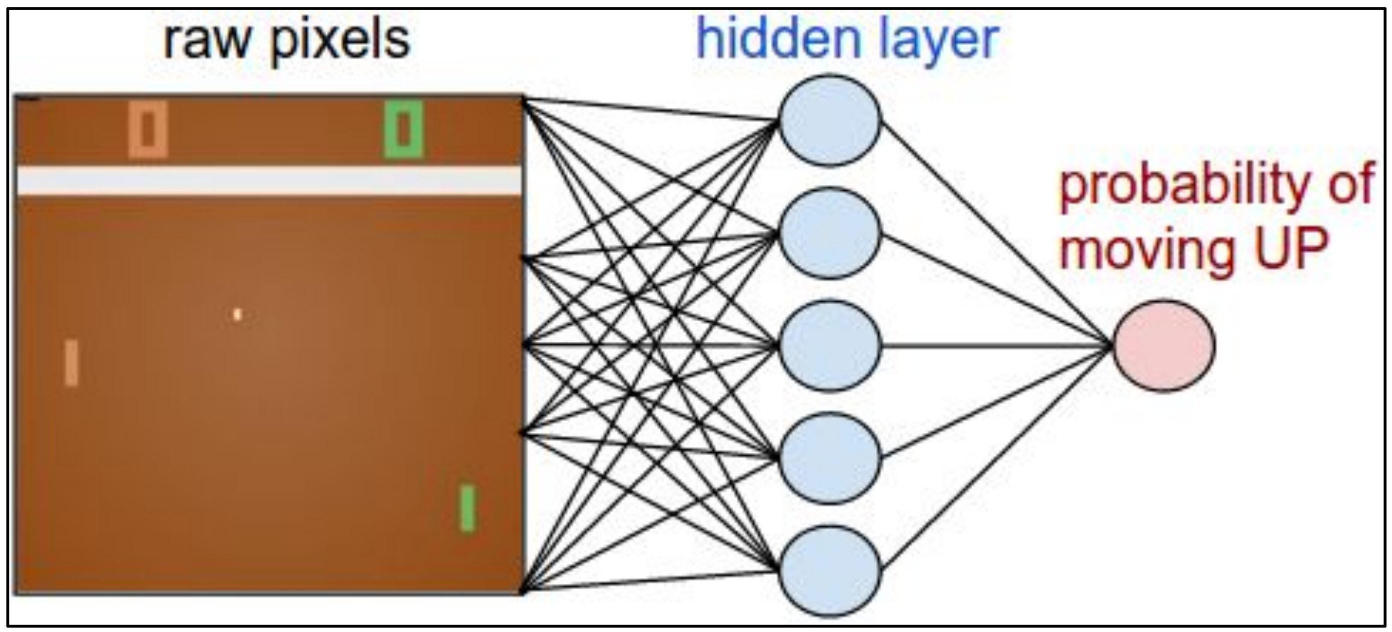
$$\mathbb{E}[\hat{g}] = \nabla_\theta U(\theta)$$

Pong from Pixels

Slides from Andrei Karpathy



Policy network

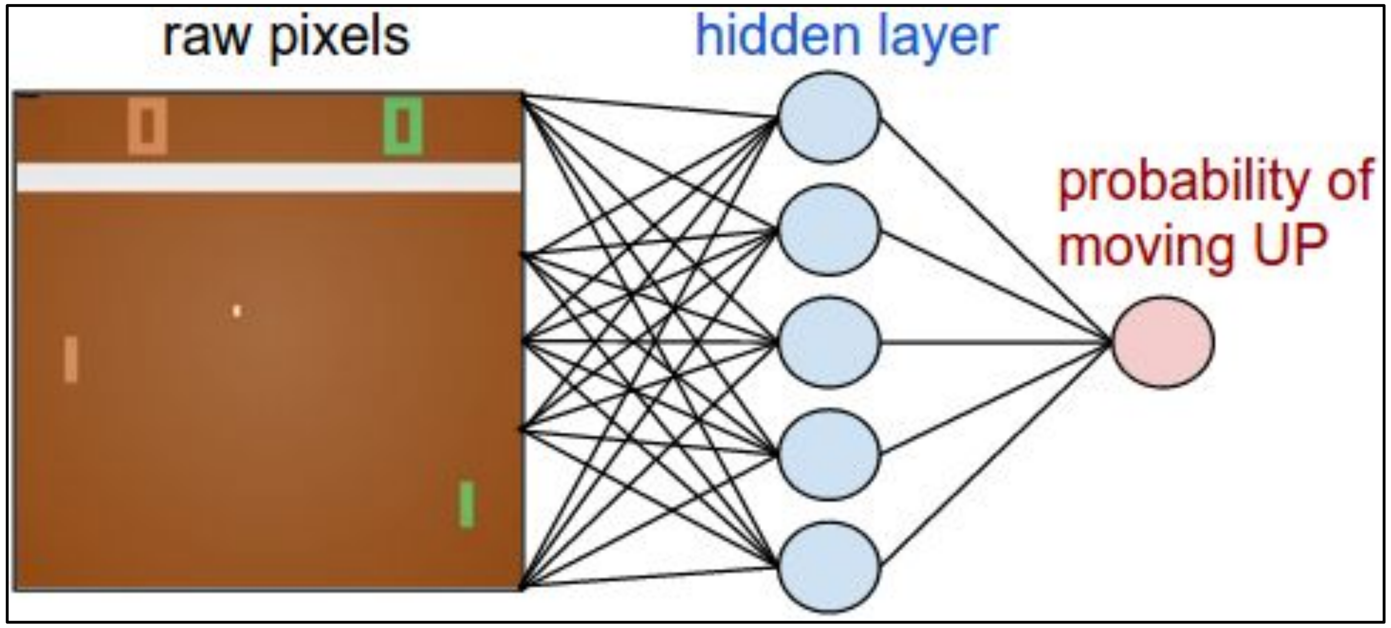


Policy network

e.g.,

height width

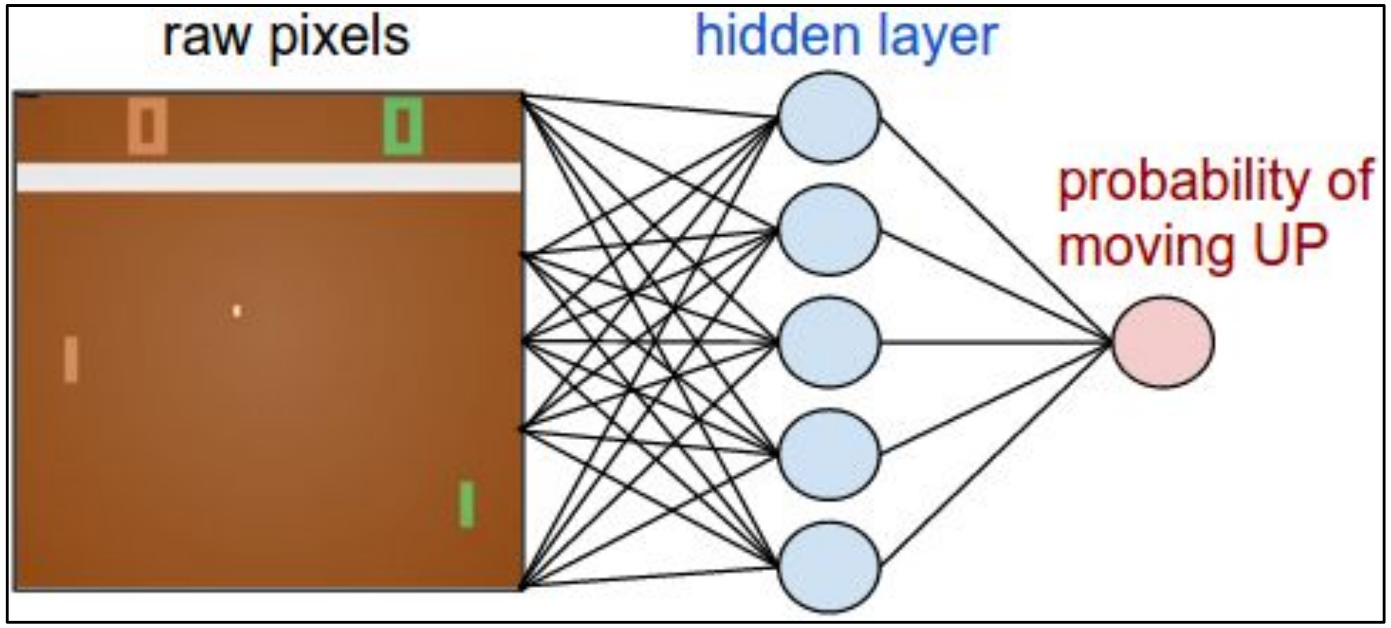
[80 x 80]
array of



Policy network

height width

[80 x 80]
array

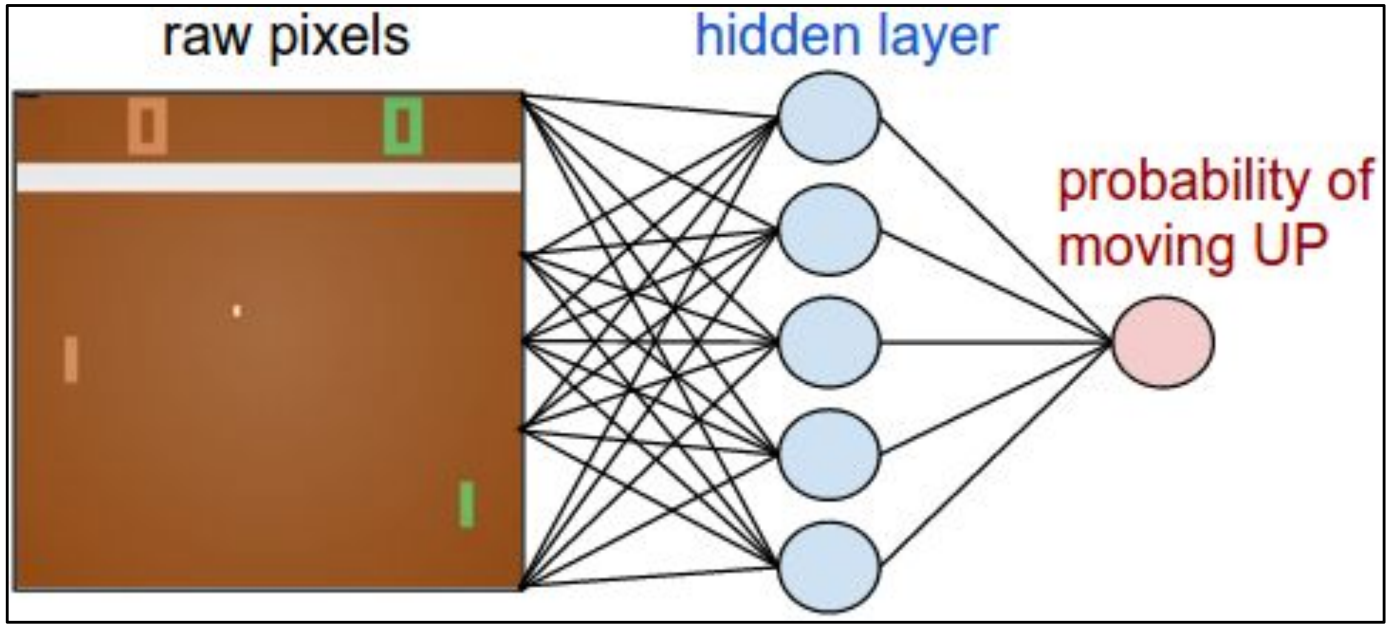



```
h = np.dot(W1, x) # compute hidden layer neuron activations
h[h<0] = 0 # ReLU nonlinearity: threshold at zero
logp = np.dot(W2, h) # compute log probability of going up
p = 1.0 / (1.0 + np.exp(-logp)) # sigmoid function (gives probability of going up)
```

Policy network

height width

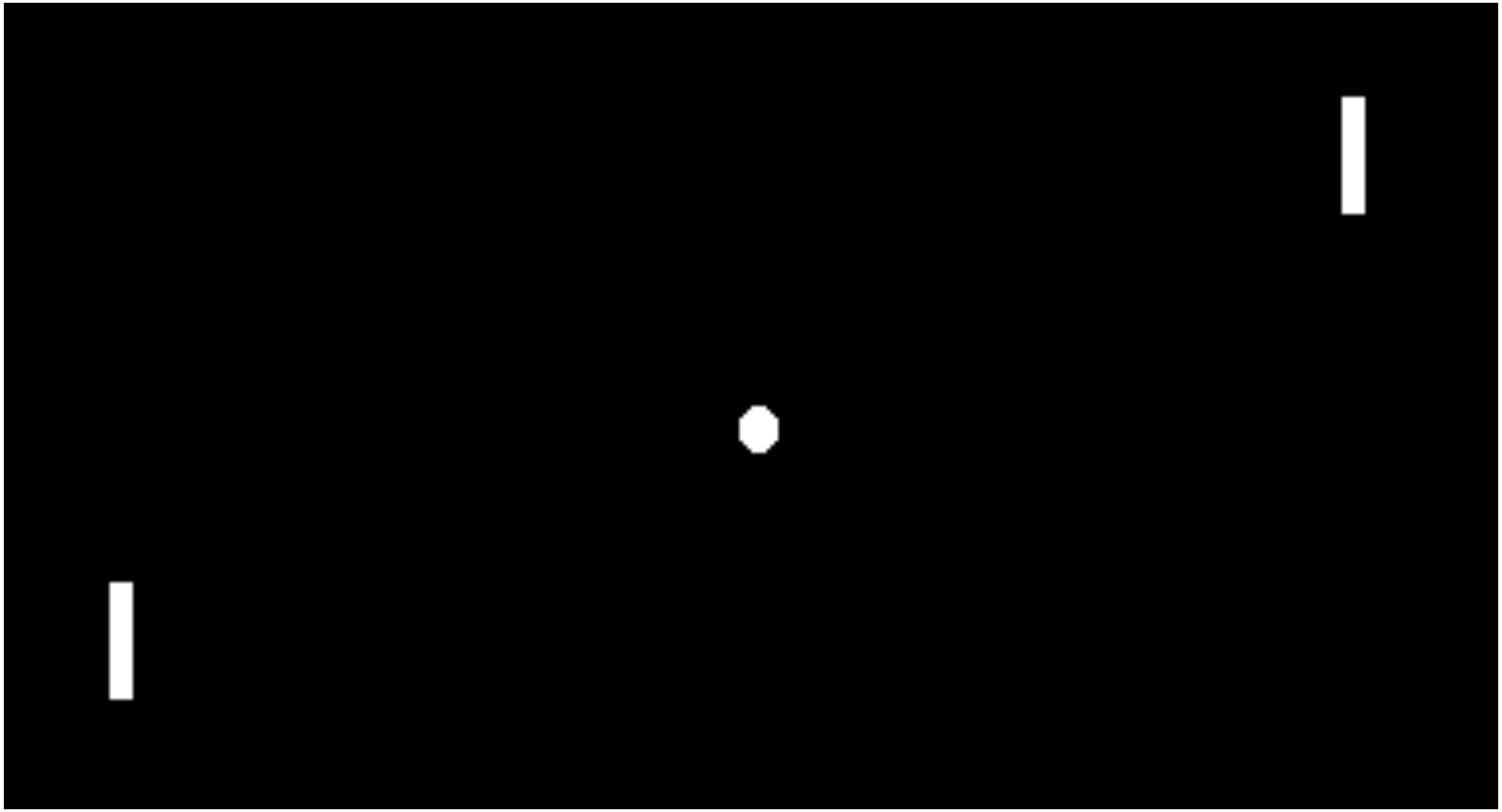
[**80** x **80**]
array



E.g. 200 nodes in the hidden network, so:

[(80*80)*200 + 200] + [200*1 + 1] = **~1.3M parameters**

Layer 1 Layer 2



Network does not see this. Network sees 80*80 = 6,400 numbers.
It gets a reward of +1 or -1, some of the time.
Q: How do we efficiently find a good setting of the 1.3M parameters?

Random search

Evolutionary methods

Approximation to the gradient via finite differences

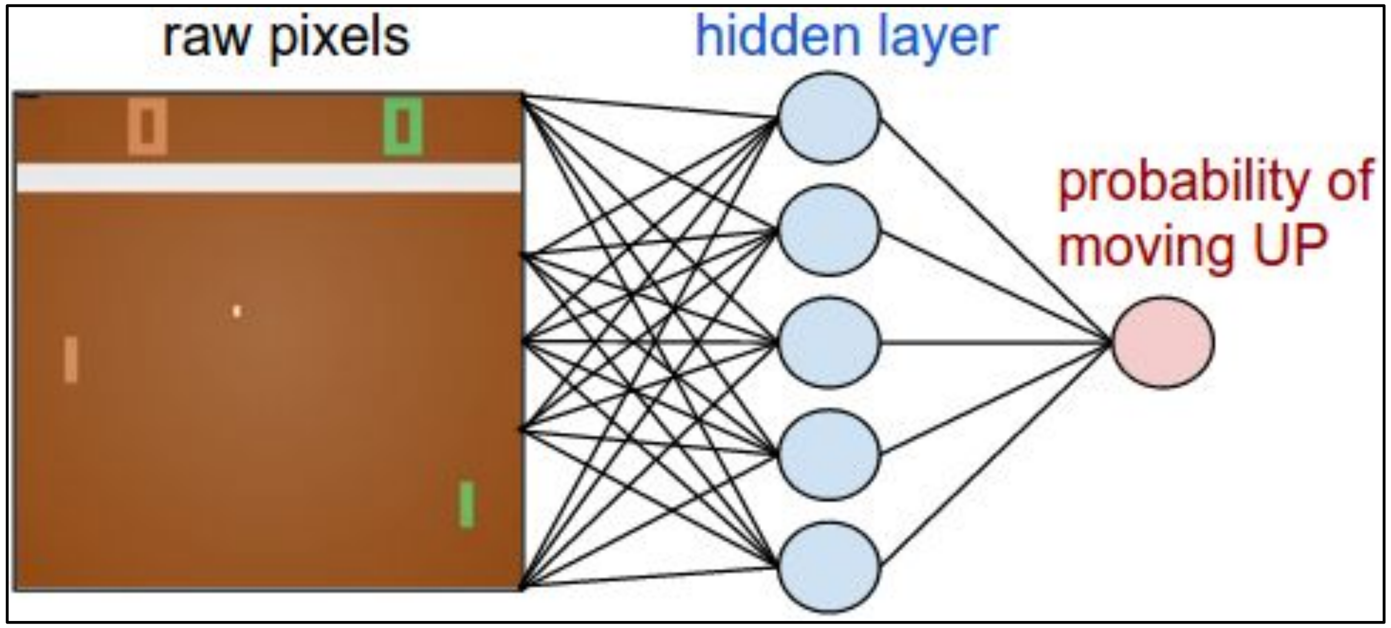
Likelihood ratio policy gradients

Suppose we had the training labels…
(we know what to do in any state)

(x1,UP)
(x2,DOWN)
(x3,UP)
...

Suppose we had the training labels…
(we know what to do in any state)

(x1,UP)
(x2,DOWN)
(x3,UP)
...

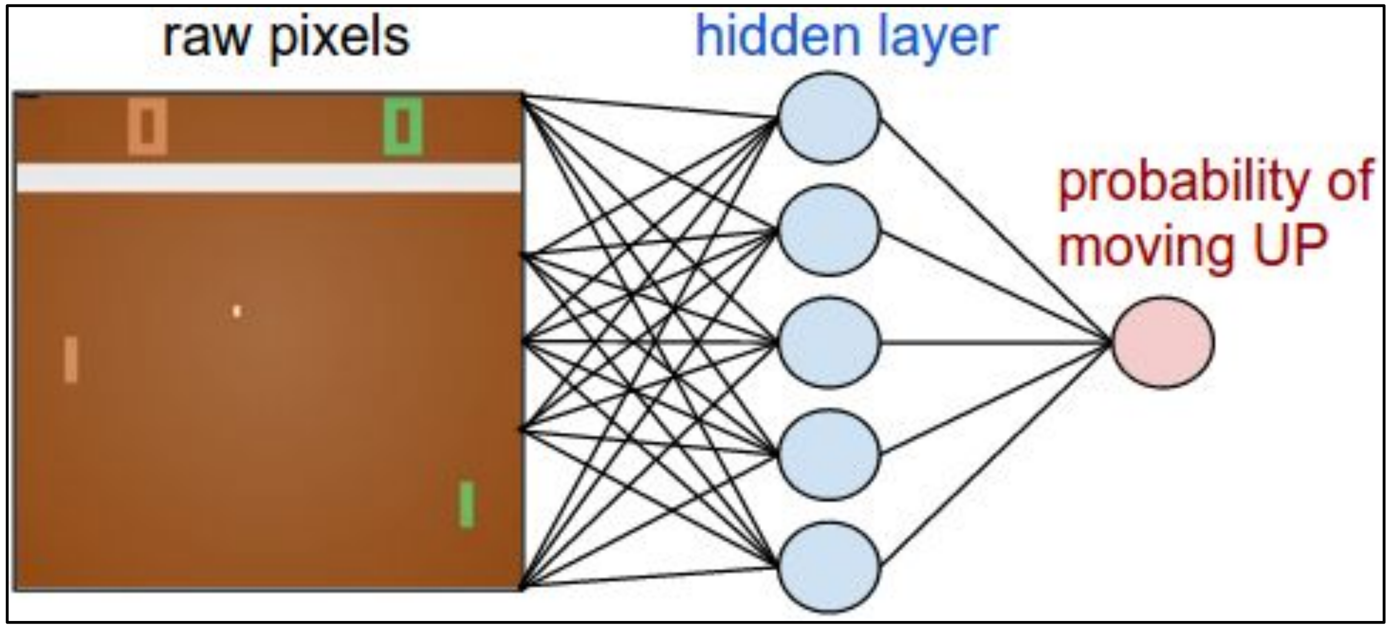


Suppose we had the training labels…
(we know what to do in any state)

(x1,UP)
(x2,DOWN)
(x3,UP)
...

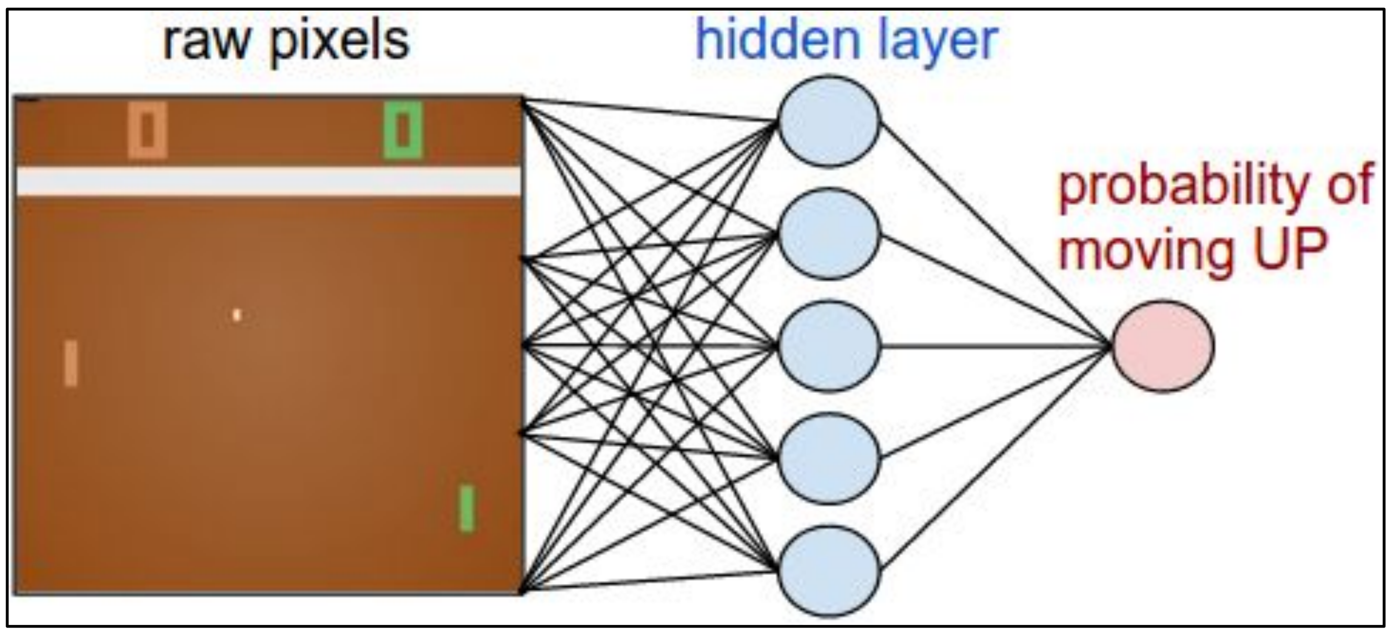
maximize:

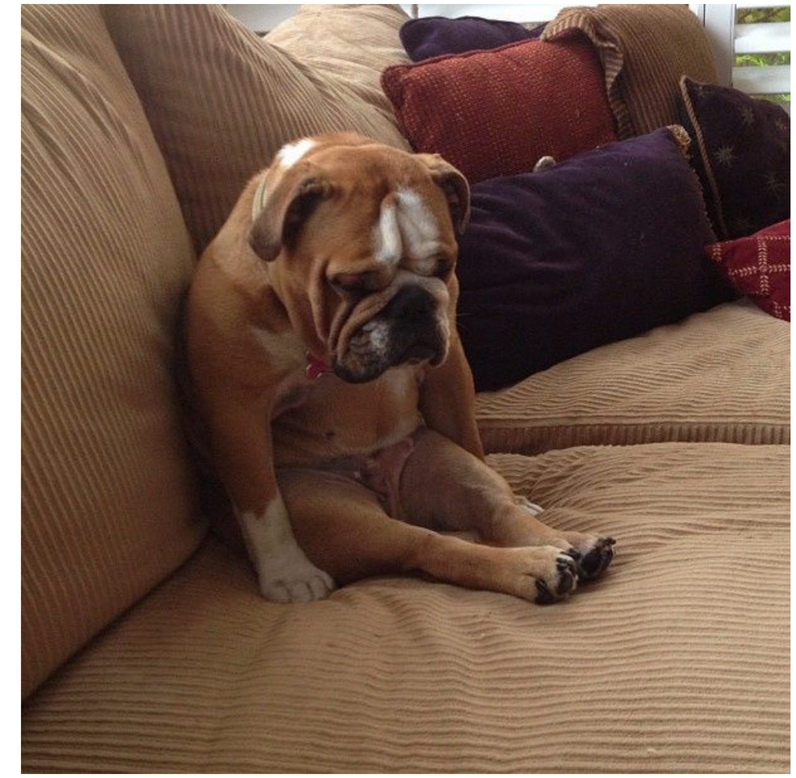
$$\boxed{\sum_i \log p(y_i \mid x_i)}$$



supervised learning

Except, we don’t have labels...





Should we go UP or DOWN?

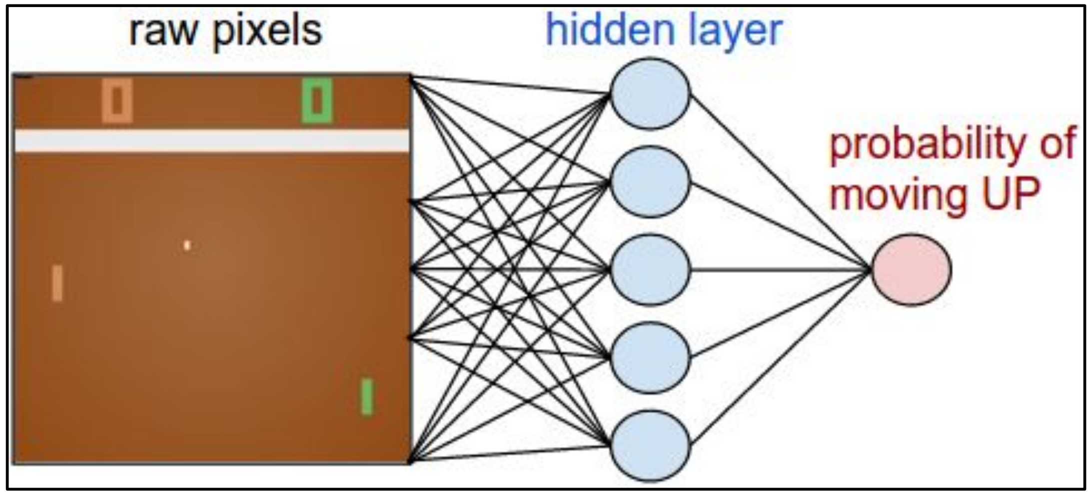
Except, we don’t have labels...

“Try a bunch of stuff and see what happens. Do more of the stuff that worked in the future.”

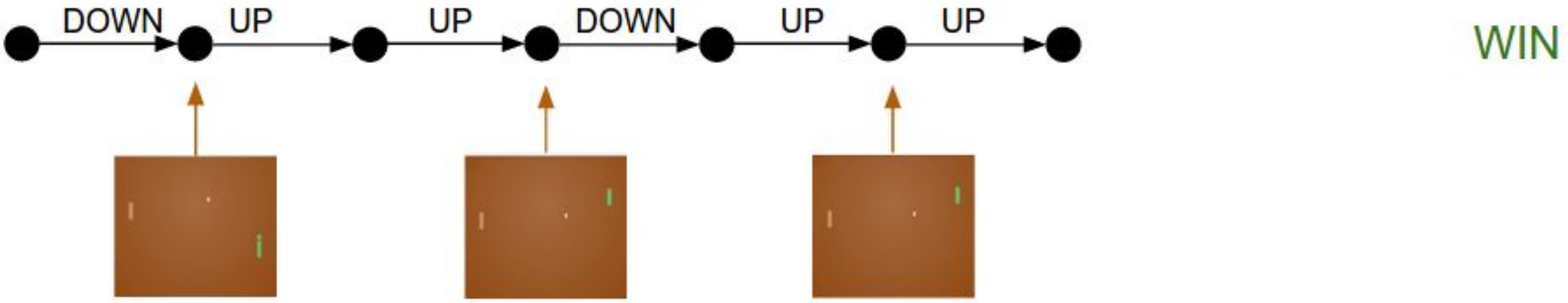
-RL

trial-and-error learning

Let's just act according to our current policy...

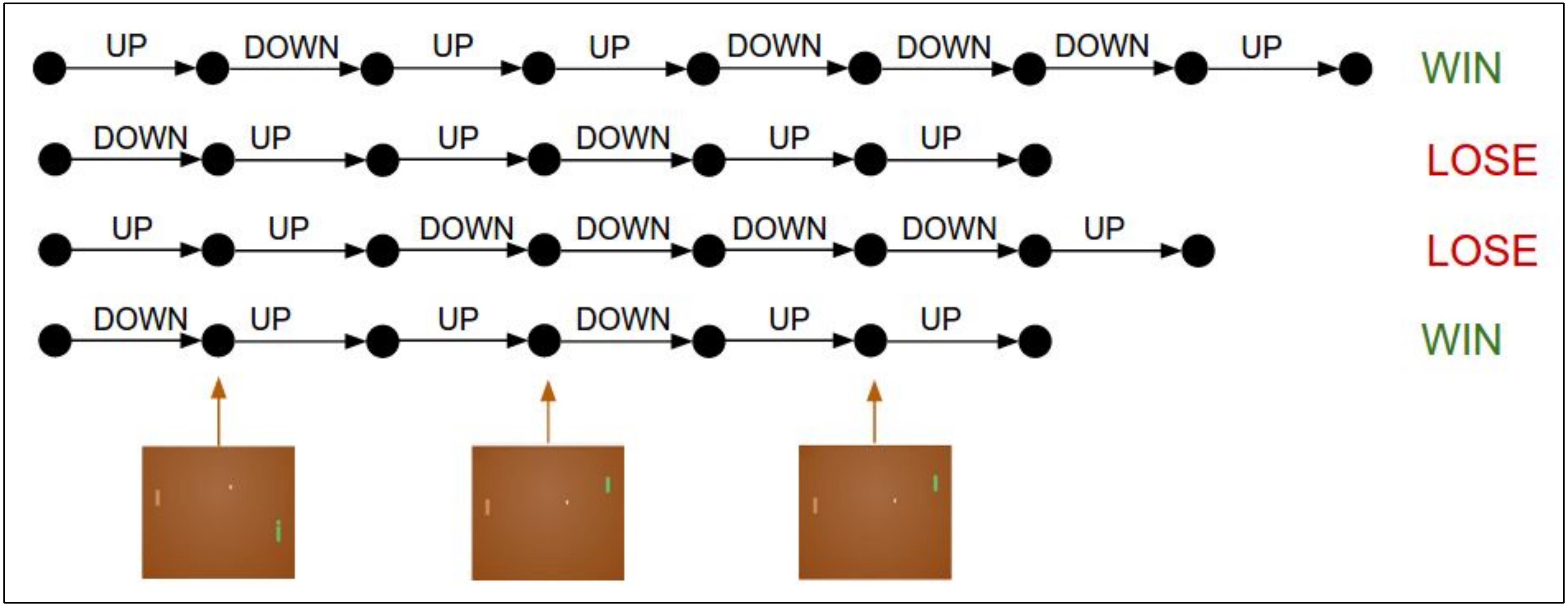


Rollout the policy and collect an episode

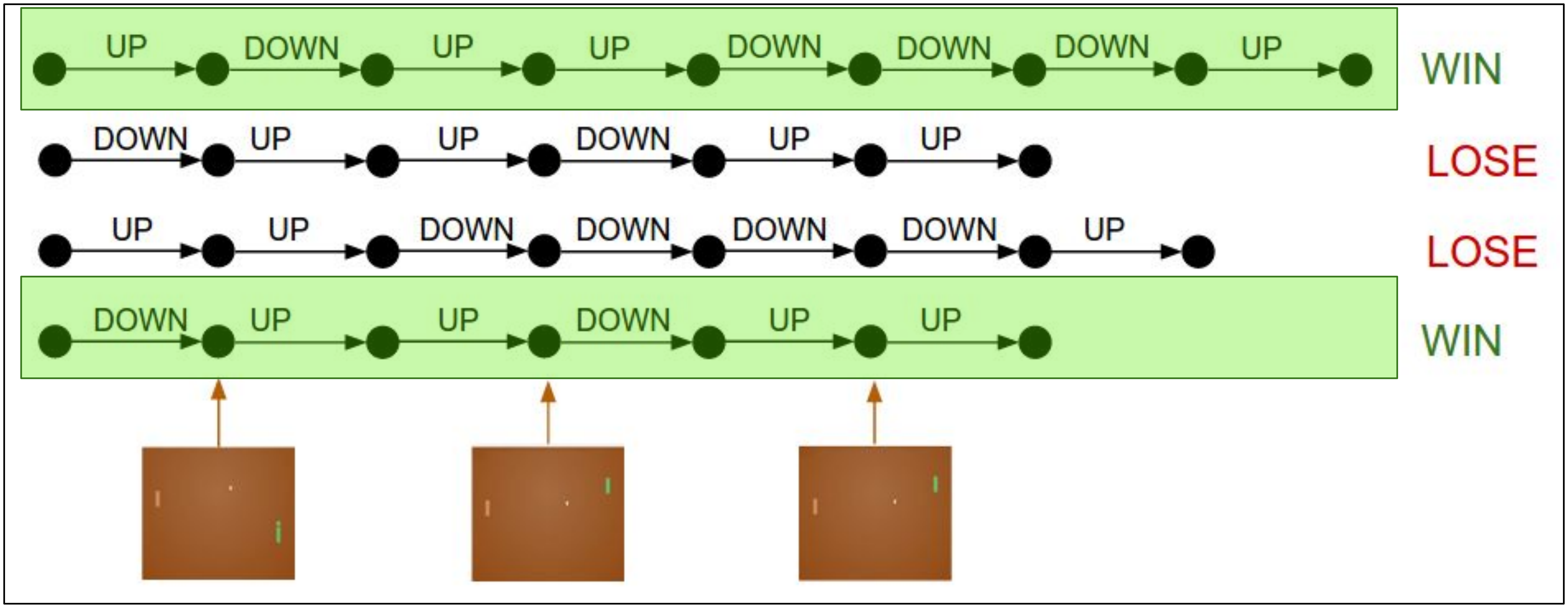


Collect many rollouts...

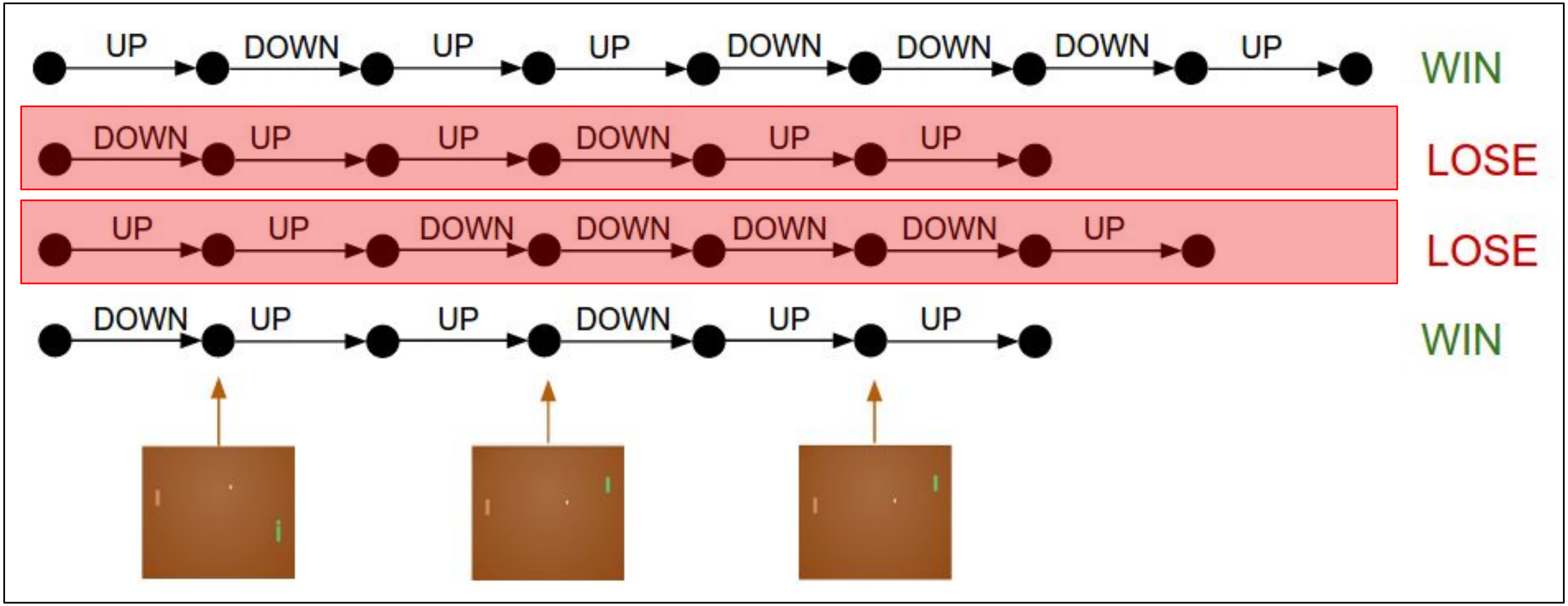
4 rollouts:



Not sure whatever we did here, but apparently it was good.



Not sure whatever we did here, but it was bad.

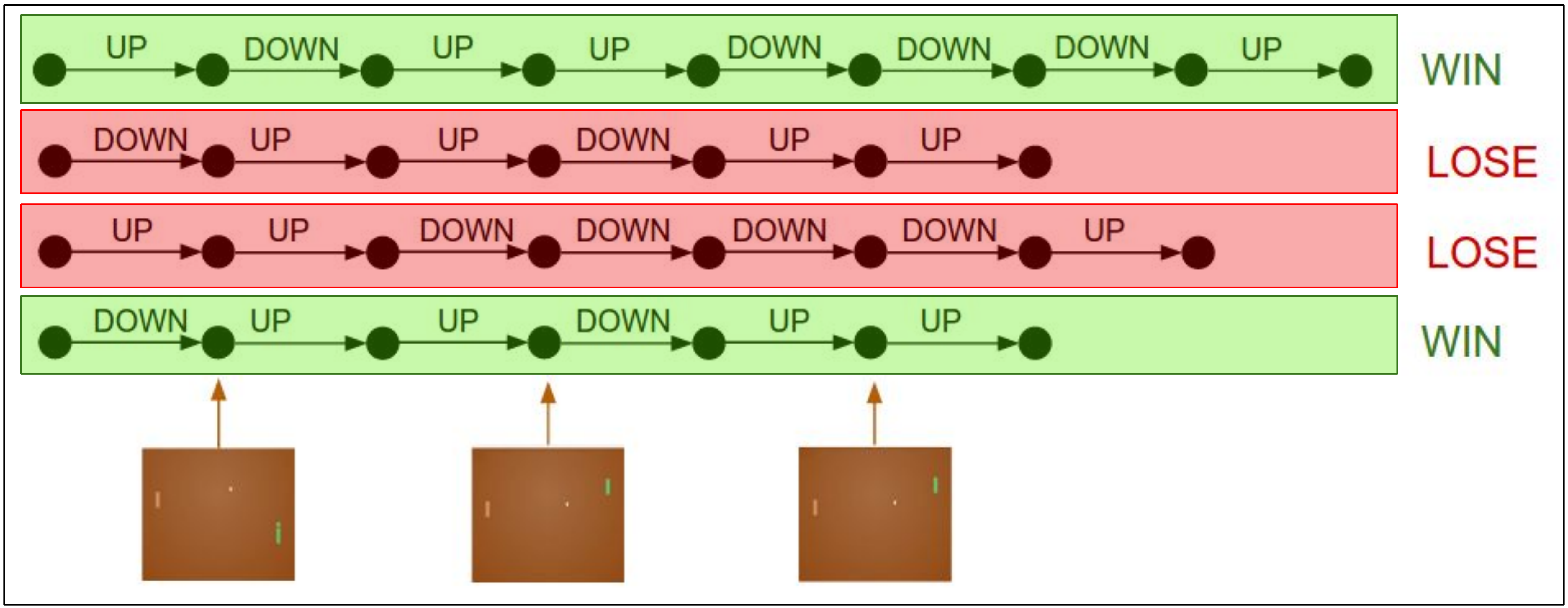


Pretend every action we took here was the correct label.

maximize: $\log p(y_i \mid x_i)$

Pretend every action we took here was the wrong label.

maximize: $(-1) * \log p(y_i \mid x_i)$



Supervised Learning

maximize:

$$\boxed{\sum_i \log p(y_i | x_i)}$$

For images x_i and their labels y_i.

Supervised Learning

maximize:

$$\boxed{\sum_i \log p(y_i | x_i)}$$

For images x_i and their labels y_i.

Reinforcement Learning

Supervised Learning

maximize:

$$\boxed{\sum_i \log p(y_i \mid x_i)}$$

For images x_i and their labels y_i.

Reinforcement Learning

1) we have no labels so we sample:

$$\boxed{y_i \sim p(\cdot \mid x_i)}$$

Supervised Learning

maximize:

$$\boxed{\sum_i \log p(y_i | x_i)}$$

For images x_i and their labels y_i.

Reinforcement Learning

1) we have no labels so we sample:

$$\boxed{y_i \sim p(\cdot | x_i)}$$

2) once we collect a batch of rollouts:

maximize:

$$\boxed{\sum_i A_i * \log p(y_i | x_i)}$$

Supervised Learning

maximize:

$$\boxed{\sum_i \log p(y_i | x_i)}$$

For images x_i and their labels y_i.

Reinforcement Learning

1) we have no labels so we sample:

$$\boxed{y_i \sim p(\cdot | x_i)}$$

2) once we collect a batch of rollouts:

maximize:

$$\boxed{\sum_i A_i * \log p(y_i | x_i)}$$

We call this the **advantage**, it's a number, like +1.0 or -1.0 based on how this action eventually turned out.

Advantage is the same for all actions taken during a trajectory, and depends on the trajectory return (episode return)

Supervised Learning

maximize:

$$\boxed{\sum_i \log p(y_i | x_i)}$$

For images x_i and their labels y_i.

Reinforcement Learning

1) we have no labels so we sample:

$$\boxed{y_i \sim p(\cdot | x_i)}$$

2) once we collect a batch of rollouts:

maximize:

$$\boxed{\sum_i A_i * \log p(y_i | x_i)}$$

+ve advantage will make that action more likely in the future, for that state.

-ve advantage will make that action less likely in the future, for that state.

Advantage is the same for all actions taken during a trajectory, and depends on the trajectory return $R(\tau)$

Temporal structure

$$\hat{g} = \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\alpha_t^{(i)} \mid s_t^{(i)})R(\tau^{(i)})$$

$$= \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\alpha_t^{(i)} \mid s_t^{(i)})\left(\sum_{k=0}^{H} R(s_k^{(i)}, a_k^{(i)})\right)$$

Each action takes the blame for the full trajectory!

Temporal structure

$$\hat{g} = \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\alpha_t^{(i)} \mid s_t^{(i)})R(\tau^{(i)})$$

$$= \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\alpha_t^{(i)} \mid s_t^{(i)})\left(\sum_{k=0}^{H} R(s_k^{(i)}, a_k^{(i)})\right)$$

$$= \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\alpha_t^{(i)} \mid s_t^{(i)})\left(\sum_{k=0}^{t-1} R(s_k^{(i)}, a_k^{(i)}) + \sum_{k=t}^{H} R(s_k^{(i)}, a_k^{(i)})\right)$$

Each action takes the blame for the full trajectory!

These rewards are not caused by actions that come after t

Temporal structure

$$\hat{g} = \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\alpha_t^{(i)} \mid s_t^{(i)})R(\tau^{(i)})$$

$$= \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\alpha_t^{(i)} \mid s_t^{(i)})\left(\sum_{k=0}^{H} R(s_k^{(i)}, a_k^{(i)})\right)$$

$$= \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\alpha_t^{(i)} \mid s_t^{(i)})\left(\sum_{k=0}^{t-1} R(s_k^{(i)}, a_k^{(i)}) + \sum_{k=t}^{H} R(s_k^{(i)}, a_k^{(i)})\right)$$

Each action takes the blame for the full trajectory!

Consider instead:

$$\hat{g} = \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\alpha_t^{(i)} \mid s_t^{(i)})\left(\sum_{k=t}^{H} R(s_k^{(i)}, a_k^{(i)})\right)$$

Each action takes the blame for the trajectory that comes after it

We can call this the return from t onwards G_t

- Let's analyze the update:

$$\Delta\theta_t = \alpha G_t \nabla_\theta \log \pi_\theta(s_t, a_t)$$

- Let's us rewrite is as follows:

$$\boldsymbol{\theta}_{t+1} \doteq \boldsymbol{\theta}_t + \alpha\gamma^t G_t \frac{\nabla_{\boldsymbol{\theta}}\pi(A_t|S_t,\boldsymbol{\theta})}{\pi(A_t|S_t,\boldsymbol{\theta})}$$

- Update is proportional to:
  - the product of a return G_t and
  - the gradient of the probability of taking the action actually taken,
  - divided by the probability of taking that action.

- Let's analyze the update:

$$\Delta\theta_t = \alpha G_t \nabla_\theta \log \pi_\theta(s_t, a_t)$$

- Let's us rewrite is as follows:

move most in the directions that favor actions that yield the highest return

$$\boldsymbol{\theta}_{t+1} \doteq \boldsymbol{\theta}_t + \alpha\gamma^t G_t \frac{\nabla_{\boldsymbol{\theta}} \pi(A_t|S_t, \boldsymbol{\theta})}{\pi(A_t|S_t, \boldsymbol{\theta})}$$

Update is inversely proportional to the action probability -- actions that are selected frequently are at an advantage (the updates will be more often in their direction)

Likelihood ratio gradient estimator

For constant b, consider this: $$\hat{g} = \frac{1}{N}\sum_{i=1}^{N} \nabla_\theta \log P_\theta(\tau^{(i)})(R(\tau^{(i)}) - b)$$

$$\hat{g} = \frac{1}{N}\sum_{i=1}^{N} \nabla_\theta \log P_\theta(\tau^{(i)}) R(\tau^{(i)}) - \frac{1}{N}\sum_{i=1}^{N} \nabla_\theta \log P_\theta(\tau^{(i)}) b$$

$$\mathbb{E}\, \nabla_\theta \log P_\theta(\tau) b$$

Likelihood ratio gradient estimator

For constant b, consider this:

$$\hat{g} = \frac{1}{N}\sum_{i=1}^{N} \nabla_\theta \log P_\theta(\tau^{(i)})(R(\tau^{(i)}) - b)$$

$$\hat{g} = \frac{1}{N}\sum_{i=1}^{N} \nabla_\theta \log P_\theta(\tau^{(i)})R(\tau^{(i)}) - \frac{1}{N}\sum_{i=1}^{N} \nabla_\theta \log P_\theta(\tau^{(i)})b$$

$$\begin{aligned}
&\mathbb{E}\, \nabla_\theta \log P_\theta(\tau) b \\
&= \sum_\tau P(\tau;\theta)\, \nabla_\theta \log P_\theta(\tau) b
\end{aligned}$$

Likelihood ratio gradient estimator

For constant b, consider this: $$\hat{g} = \frac{1}{N}\sum_{i=1}^{N} \nabla_\theta \log P_\theta(\tau^{(i)})(R(\tau^{(i)}) - b)$$

$$\hat{g} = \frac{1}{N}\sum_{i=1}^{N} \nabla_\theta \log P_\theta(\tau^{(i)})R(\tau^{(i)}) - \frac{1}{N}\sum_{i=1}^{N} \nabla_\theta \log P_\theta(\tau^{(i)})b$$

$$\begin{aligned}
&\mathbb{E}\,\nabla_\theta \log P_\theta(\tau)b \\
&= \sum_\tau P(\tau;\theta)\,\nabla_\theta \log P_\theta(\tau)b \\
&= \sum_\tau P(\tau;\theta)\frac{\nabla_\theta P_\theta(\tau)}{P(\tau;\theta)}b
\end{aligned}$$

Likelihood ratio gradient estimator

For constant b, consider this:

$$\hat{g} = \frac{1}{N}\sum_{i=1}^{N} \nabla_\theta \log P_\theta(\tau^{(i)})(R(\tau^{(i)}) - b)$$

$$\hat{g} = \frac{1}{N}\sum_{i=1}^{N} \nabla_\theta \log P_\theta(\tau^{(i)})R(\tau^{(i)}) - \frac{1}{N}\sum_{i=1}^{N} \nabla_\theta \log P_\theta(\tau^{(i)})b$$

$$\begin{aligned}
&\mathbb{E}\,\nabla_\theta \log P_\theta(\tau)b \\
&= \sum_\tau P(\tau;\theta)\,\nabla_\theta \log P_\theta(\tau)b \\
&= \sum_\tau P(\tau;\theta)\frac{\nabla_\theta P_\theta(\tau)}{P(\tau;\theta)}b \\
&= \sum_\tau \nabla_\theta P_\theta(\tau)b
\end{aligned}$$

Likelihood ratio gradient estimator

For constant b, consider this: $$\hat{g} = \frac{1}{N}\sum_{i=1}^{N} \nabla_\theta \log P_\theta(\tau^{(i)})(R(\tau^{(i)}) - b)$$

$$\hat{g} = \frac{1}{N}\sum_{i=1}^{N} \nabla_\theta \log P_\theta(\tau^{(i)})R(\tau^{(i)}) - \frac{1}{N}\sum_{i=1}^{N} \nabla_\theta \log P_\theta(\tau^{(i)})b$$

$$\begin{aligned}
&\mathbb{E}\, \nabla_\theta \log P_\theta(\tau) b \\
&= \sum_\tau P(\tau;\theta)\, \nabla_\theta \log P_\theta(\tau) b \\
&= \sum_\tau P(\tau;\theta) \frac{\nabla_\theta P_\theta(\tau)}{P(\tau;\theta)} b \\
&= \sum_\tau \nabla_\theta P_\theta(\tau) b \\
&= b \left(\sum_\tau \nabla_\theta P_\theta(\tau) \right)
\end{aligned}$$

Likelihood ratio gradient estimator

For constant b, consider this: $$\hat{g} = \frac{1}{N}\sum_{i=1}^{N} \nabla_\theta \log P_\theta(\tau^{(i)})(R(\tau^{(i)}) - b)$$

$$\hat{g} = \frac{1}{N}\sum_{i=1}^{N} \nabla_\theta \log P_\theta(\tau^{(i)})R(\tau^{(i)}) - \frac{1}{N}\sum_{i=1}^{N} \nabla_\theta \log P_\theta(\tau^{(i)})b$$

$$\begin{aligned}
&\mathbb{E}\,\nabla_\theta \log P_\theta(\tau)b \\
&= \sum_\tau P(\tau;\theta)\,\nabla_\theta \log P_\theta(\tau)b \\
&= \sum_\tau P(\tau;\theta)\frac{\nabla_\theta P_\theta(\tau)}{P(\tau;\theta)}b \\
&= \sum_\tau \nabla_\theta P_\theta(\tau)b \\
&= b\left(\sum_\tau \nabla_\theta P_\theta(\tau)\right) \\
&= b\left(\nabla_\theta \sum_\tau P_\theta(\tau)\right)
\end{aligned}$$

Likelihood ratio gradient estimator

For constant b, consider this: $$\hat{g} = \frac{1}{N}\sum_{i=1}^{N} \nabla_\theta \log P_\theta(\tau^{(i)})(R(\tau^{(i)}) - b)$$

$$\hat{g} = \frac{1}{N}\sum_{i=1}^{N} \nabla_\theta \log P_\theta(\tau^{(i)})R(\tau^{(i)}) - \frac{1}{N}\sum_{i=1}^{N} \nabla_\theta \log P_\theta(\tau^{(i)})b$$

$$\begin{aligned}
&\mathbb{E}\,\nabla_\theta \log P_\theta(\tau)b \\
&= \sum_\tau P(\tau;\theta)\,\nabla_\theta \log P_\theta(\tau)b \\
&= \sum_\tau P(\tau;\theta)\frac{\nabla_\theta P_\theta(\tau)}{P(\tau;\theta)}b \\
&= \sum_\tau \nabla_\theta P_\theta(\tau)b \\
&= b\left(\sum_\tau \nabla_\theta P_\theta(\tau)\right) \\
&= b\left(\nabla_\theta \sum_\tau P_\theta(\tau)\right) \\
&= 0
\end{aligned}$$

We still have an unbiased estimator of the gradient!

Baseline choices

$$\hat{g} = \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\alpha_t^{(i)} \mid s_t^{(i)})\left(\sum_{k=t}^{H} R(s_k^{(i)}, a_k^{(i)}) - b\right)$$

- Constant baseline: $b = \mathbb{E}[R(\tau)] \approx \sum_{i=1}^{N} R(\tau^{(i)})$
- Time-dependent baseline: $b_t = \sum_{i=1}^{N}\sum_{k=t}^{H} R(s_k^{(i)}, a_k^{(i)})$
- State-dependent expected return:

$$b(s_t) = \mathbb{E}\left[r_t + r_{t+1} + r_{t+2} + \ldots + r_{H-1}\right] = V_\pi(s_t)$$

Estimate $V_\pi(s_t)$

$$\hat{g} = \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\alpha_t^{(i)} \mid s_t^{(i)})\left(\sum_{k=t}^{H} R(s_k^{(i)}, a_k^{(i)}) - V^\pi(s_k^{(i)})\right)$$

MC estimation

Initialize ϕ

- Collect trajectories $\tau_1, \ldots \tau_N$
- Regress against empirical return:

$$\phi_{i+1} \leftarrow \arg\min_{\phi} \frac{1}{N}\sum_{i=1}^{N}\sum_{t=0}^{H-1}\left(V_\phi^\pi(s_t^{(i)}) - (\sum_{k=t}^{H-1} R(s_k^{(i)}, u_k^{(i)}))\right)^2$$

REINFORCE

Algorithm 1 "Vanilla" policy gradient algorithm

Initialize policy parameter θ, baseline b

for iteration$=1, 2, \ldots$ **do**

  Collect a set of trajectories by executing the current policy

  At each timestep in each trajectory, compute

   the *return* $R_t = \sum_{t'=t}^{T-1} \gamma^{t'-t} r_{t'}$, and

   the *advantage estimate* $\hat{A}_t = R_t - b(s_t)$.

  Re-fit the baseline, by minimizing $\|b(s_t) - R_t\|^2$,

   summed over all trajectories and timesteps.

  Update the policy, using a policy gradient estimate $\hat{g}$,

   which is a sum of terms $\nabla_\theta \log \pi(a_t \mid s_t, \theta)\hat{A}_t$

end for

~ [Williams, 1992]

Estimate $V_\pi(s_t)$

$$\hat{g} = \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\alpha_t^{(i)} \mid s_t^{(i)})\left(\sum_{k=t}^{H} R(s_k^{(i)}, a_k^{(i)}) - V^\pi(s_k^{(i)})\right)$$

TD estimation

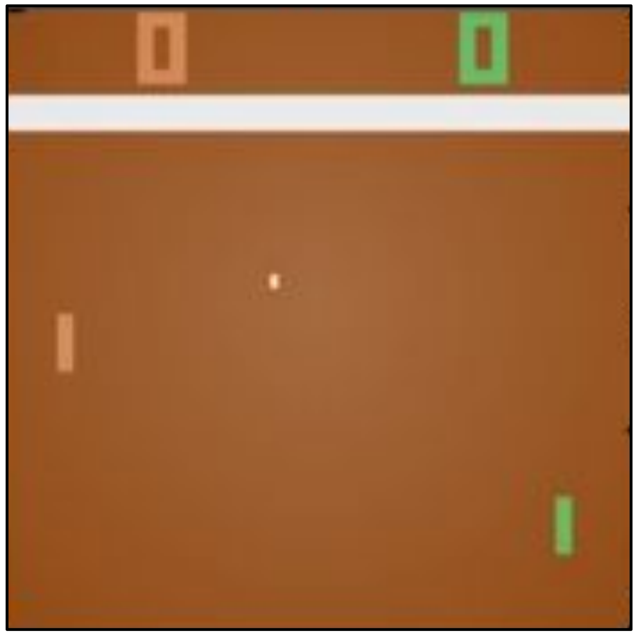
Initialize ϕ

- Collect data {s, u, s', r}
- Fitted V iteration:

$$\phi_{i+1} \leftarrow \min_{\phi} \sum_{(s,u,s',r)} \|r + V^\pi_{\phi_i}(s') - V_\phi(s)\|_2^2 + \lambda\|\phi - \phi_i\|_2^2$$

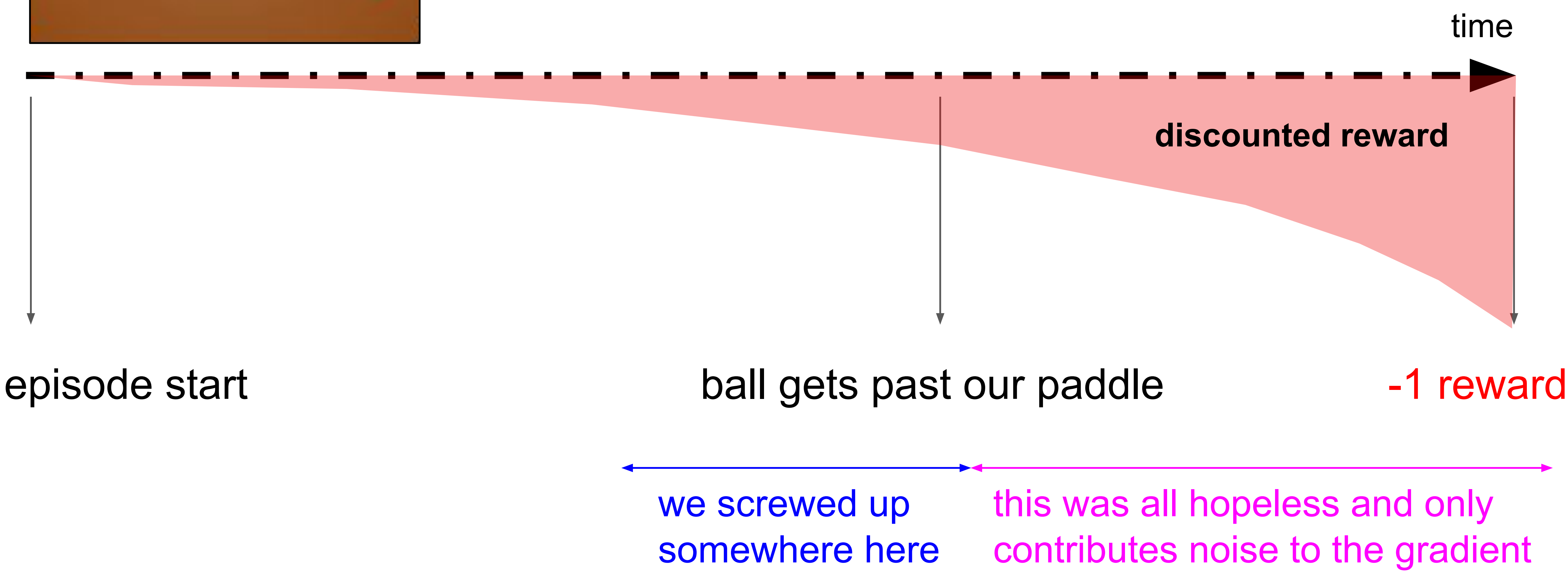
Bootstrapping!

Actions inherit the blame of the future return



All the random actions we did have been found bad, while they really didn't matter..

Can I find a better estimator for the cumulative future reward, instead of the return of a single rollout?



Better estimates for cumulative future reward

$$\hat{g} = \frac{1}{N}\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_\theta \log \pi_\theta(\alpha_t^{(i)} \mid s_t^{(i)})\left(\sum_{k=t}^{H} R(s_k^{(i)}, a_k^{(i)}) - V^\pi(s_k^{(i)})\right)$$

- Estimation of Q from *single* roll-out

$$Q^\pi(s,u) = \mathbb{E}[r_0 + r_1 + r_2 + \cdots | s_0 = s, a_0 = a]$$

- = high variance per sample based / no generalization
  - Reduce variance by discounting
  - Reduce variance by function approximation (=critic)

Actor-Critic

- Monte-Carlo policy gradient still has high variance
- We can use a critic to estimate the action-value function:

$$Q_w(s, a) \approx Q^{\pi_\theta}(s, a)$$

- Actor-critic algorithms maintain two sets of parameters
  - Critic Updates action-value function parameters w
  - Actor Updates policy parameters θ, in direction suggested by critic
- Actor-critic algorithms follow an approximate policy gradient

Actor-Critic

$$Q^{\pi,\gamma}(s,u) = \mathbb{E}[r_0 + \gamma r_1 + \gamma^2 r_2 + \cdots \mid s_0 = s, u_0 = u]$$

Actor-Critic

$$
\begin{aligned}
Q^{\pi,\gamma}(s,u) &= \mathbb{E}[r_0 + \gamma r_1 + \gamma^2 r_2 + \cdots \mid s_0 = s, u_0 = u] \\
&= \mathbb{E}[r_0 + \gamma V^{\pi}(s_1) \mid s_0 = s, u_0 = u]
\end{aligned}
$$

Actor-Critic

$$
\begin{aligned}
Q^{\pi,\gamma}(s,u) &= \mathbb{E}[r_0 + \gamma r_1 + \gamma^2 r_2 + \cdots \mid s_0 = s, u_0 = u] \\
&= \mathbb{E}[r_0 + \gamma V^{\pi}(s_1) \mid s_0 = s, u_0 = u] \\
&= \mathbb{E}[r_0 + \gamma r_1 + \gamma^2 V^{\pi}(s_2) \mid s_0 = s, u_0 = u]
\end{aligned}
$$

Actor-Critic

$$\begin{aligned}
Q^{\pi,\gamma}(s,u) &= \mathbb{E}[r_0 + \gamma r_1 + \gamma^2 r_2 + \cdots \mid s_0 = s, u_0 = u] \\
&= \mathbb{E}[r_0 + \gamma V^\pi(s_1) \mid s_0 = s, u_0 = u] \\
&= \mathbb{E}[r_0 + \gamma r_1 + \gamma^2 V^\pi(s_2) \mid s_0 = s, u_0 = u] \\
&= \mathbb{E}[r_0 + \gamma r_1 + + \gamma^2 r_2 + \gamma^3 V^\pi(s_3) \mid s_0 = s, u_0 = u] \\
&= \cdots
\end{aligned}$$

- ***Async Advantage Actor Critic (A3C)*** *[Mnih et al, 2016]*
  - $\hat{Q}$ one of the above choices (e.g. k=5 step lookahead)

Asynchronous Deep RL

Asynchronous Methods for Deep Reinforcement Learning

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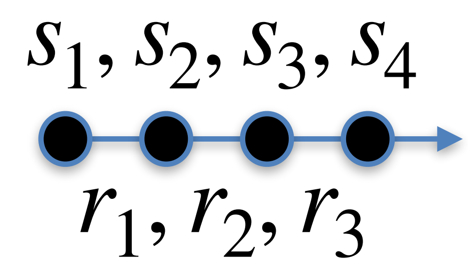
Algorithm S3 Asynchronous advantage actor-critic - pseudocode for each actor-learner thread.

// Assume global shared parameter vectors θ and θ_v and global shared counter $T = 0$
// Assume thread-specific parameter vectors θ' and θ'_v
Initialize thread step counter $t \leftarrow 1$
repeat
    Reset gradients: $d\theta \leftarrow 0$ and $d\theta_v \leftarrow 0$.
    Synchronize thread-specific parameters $\theta' = \theta$ and $\theta'_v = \theta_v$
    $t_{start} = t$
    Get state s_t
    repeat
        Perform a_t according to policy $\pi(a_t|s_t;\theta')$
        Receive reward r_t and new state s_{t+1}
        $t \leftarrow t + 1$
        $T \leftarrow T + 1$
    until terminal s_t **or** $t - t_{start} == t_{max}$
    $R = \begin{cases} 0 & \text{for terminal } s_t \\ V(s_t, \theta'_v) & \text{for non-terminal } s_t \text{// Bootstrap from last state} \end{cases}$
    for $i \in \{t-1, \ldots, t_{start}\}$ **do**
        $R \leftarrow r_i + \gamma R$
        Accumulate gradients wrt θ': $d\theta \leftarrow d\theta + \nabla_{\theta'} \log \pi(a_i|s_i;\theta')(R - V(s_i;\theta'_v))$
        Accumulate gradients wrt θ'_v: $d\theta_v \leftarrow d\theta_v + \partial\left(R - V(s_i;\theta'_v)\right)^2 / \partial\theta'_v$
    end for
    Perform asynchronous update of θ using $d\theta$ and of θ_v using $d\theta_v$.
until $T > T_{max}$

A3C

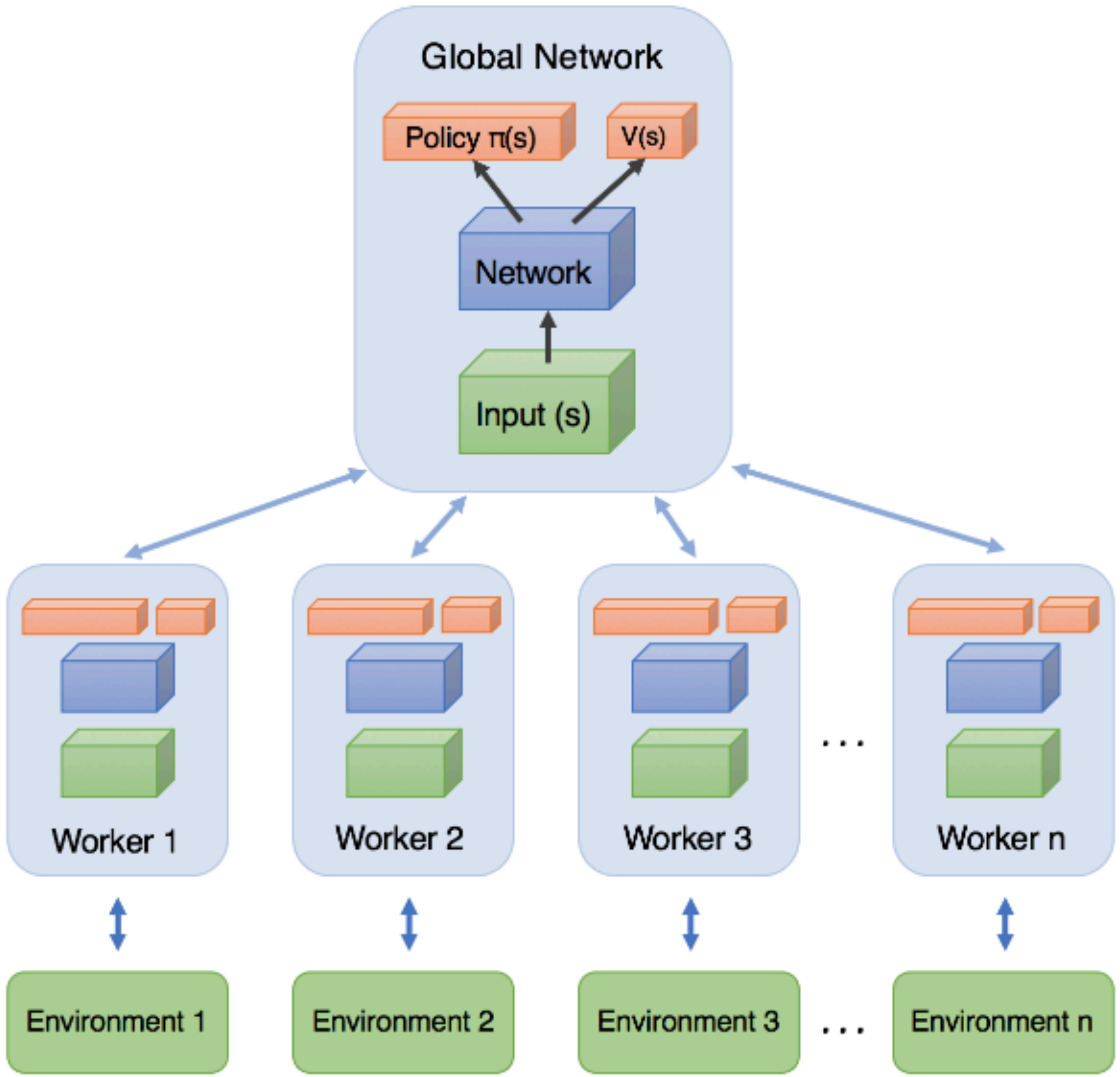


What is the approximation used for the advantage?

$R_3 = r_3 + \gamma V(s_4, \theta'_v)$ $\quad$ $A_3 = R_3 - V(s_3;\theta'_v)$

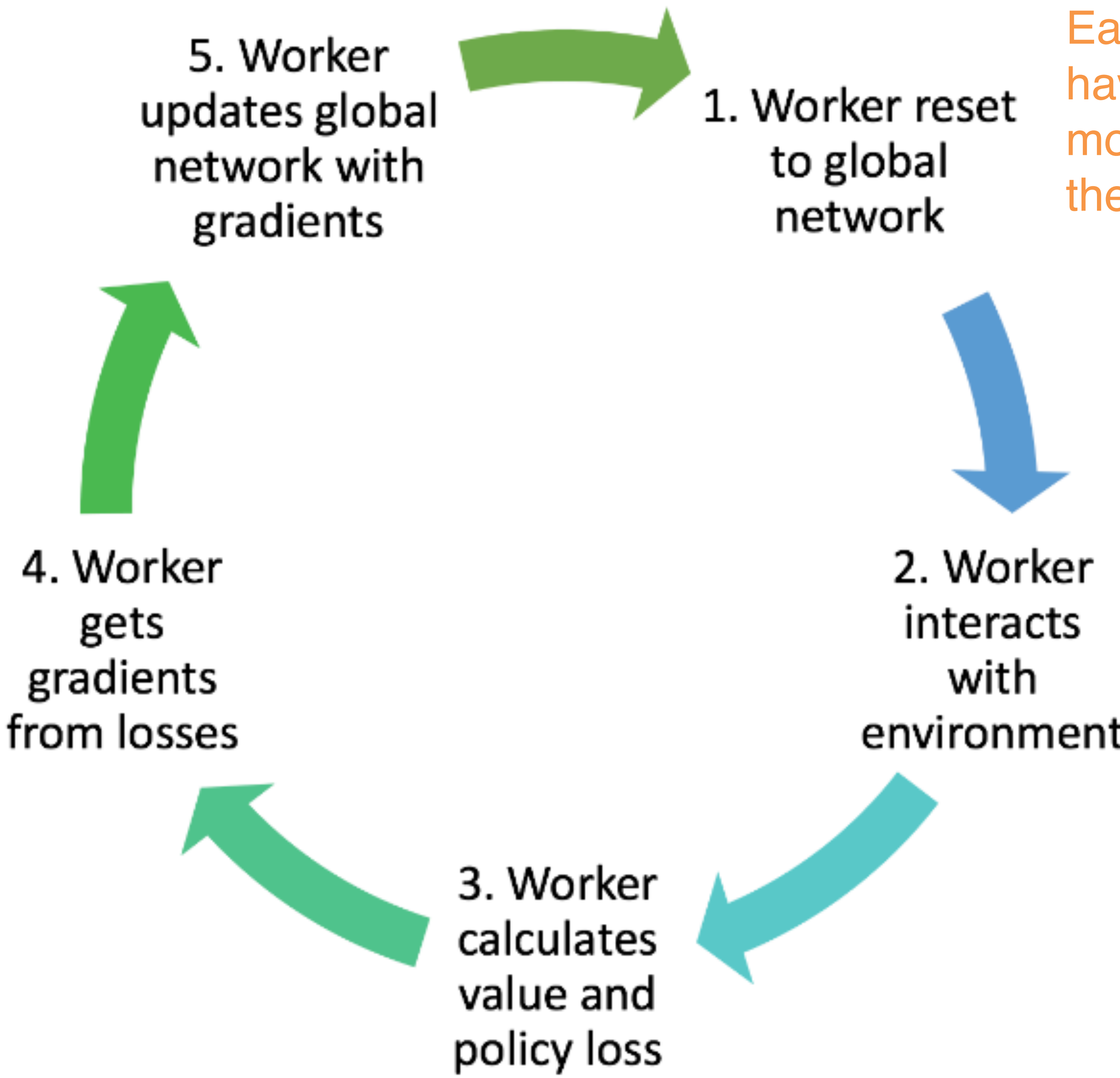
$R_2 = r_2 + \gamma r_3 + \gamma^2 V(s_4, \theta'_v)$ $\quad$ $A_2 = R_2 - V(s_2;\theta'_v)$

Distributed RL



Distributed Asynchronous RL

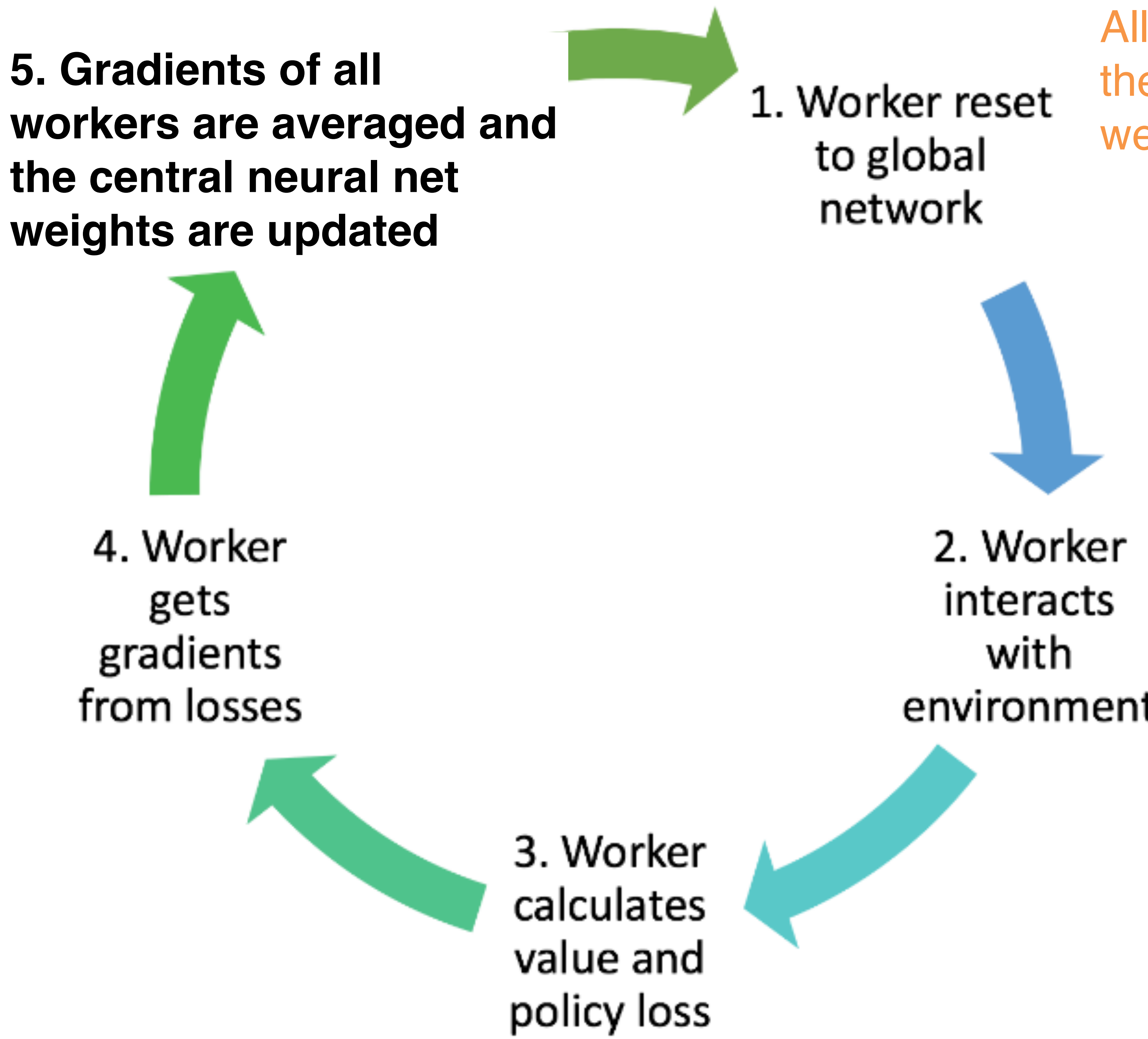
No locking



Each worker may have slightly modified version of the policy/critic

The actor critic trained in such asynchronous way is knows as A3C

Distributed Synchronous RL



All worker may have the same actor/critic weights

The actor critic trained in such synchronous way is knows as A2C

Advantages of Asynchronous (multi-threaded) RL