Carnegie Mellon

School of Computer Science

Deep Reinforcement Learning and Control

Temporal Difference Learning

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Used Materials

- **Disclaimer**: Much of the material and slides for this lecture were borrowed from Rich Sutton's class and David Silver's class on Reinforcement Learning.

MC and TD Learning

- Goal: learn $v_\pi(s)$ from episodes of experience under policy π

- Incremental every-visit Monte-Carlo:
  - Update value $V(S_t)$ toward actual return G_t

$$V(S_t) \leftarrow V(S_t) + \alpha \left(G_t - V(S_t) \right)$$

- Simplest Temporal-Difference learning algorithm: TD(0)
  - Update value $V(S_t)$ toward estimated returns $R_{t+1} + \gamma V(S_{t+1})$

$$V(S_t) \leftarrow V(S_t) + \alpha \left(R_{t+1} + \gamma V(S_{t+1}) - V(S_t) \right)$$

- $R_{t+1} + \gamma V(S_{t+1})$ is called the TD target
- $\delta_t = R_{t+1} + \gamma V(S_{t+1}) - V(S_t)$ called the TD error.

DP vs. MC vs. TD Learning

- Remember:

MC: sample average return approximates expectation

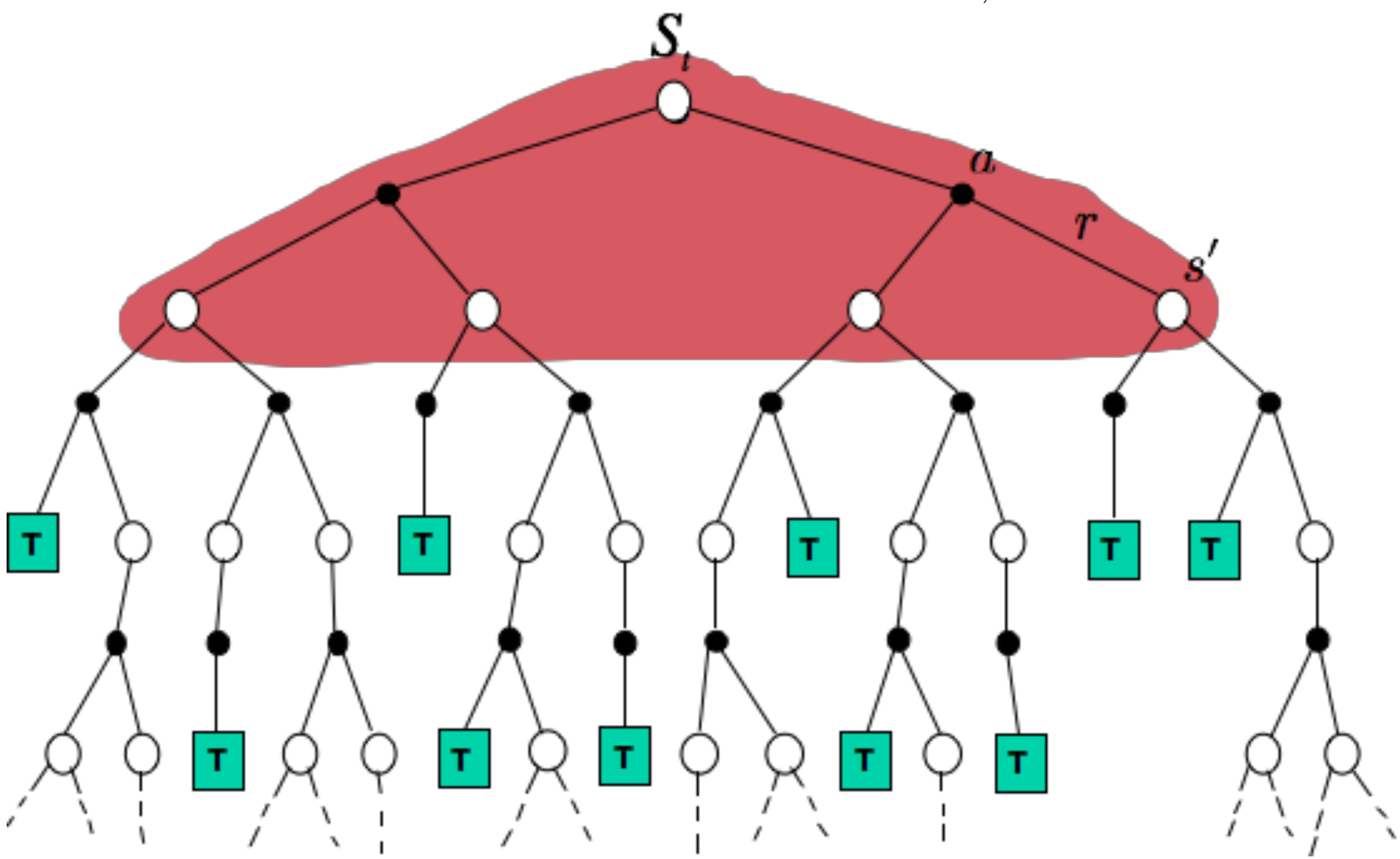
$$
\begin{aligned}
v_\pi(s) &\doteq \mathbb{E}_\pi[G_t \mid S_t = s] \\
&= \mathbb{E}_\pi\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \,\middle|\, S_t = s\right] \\
&= \mathbb{E}_\pi\left[R_{t+1} + \gamma \sum_{k=0}^{\infty} \gamma^k R_{t+k+2} \,\middle|\, S_t = s\right] \\
&= \mathbb{E}_\pi[R_{t+1} + \gamma v_\pi(S_{t+1}) \mid S_t = s] \,.
\end{aligned}
$$

TD: combine both: Sample expected values *and* use a current estimate $V(S_{t+1})$ of the true $v_\pi(S_{t+1})$

DP: the expected values are provided by a model. But we use a current estimate $V(S_{t+1})$ of the true $v_\pi(S_{t+1})$

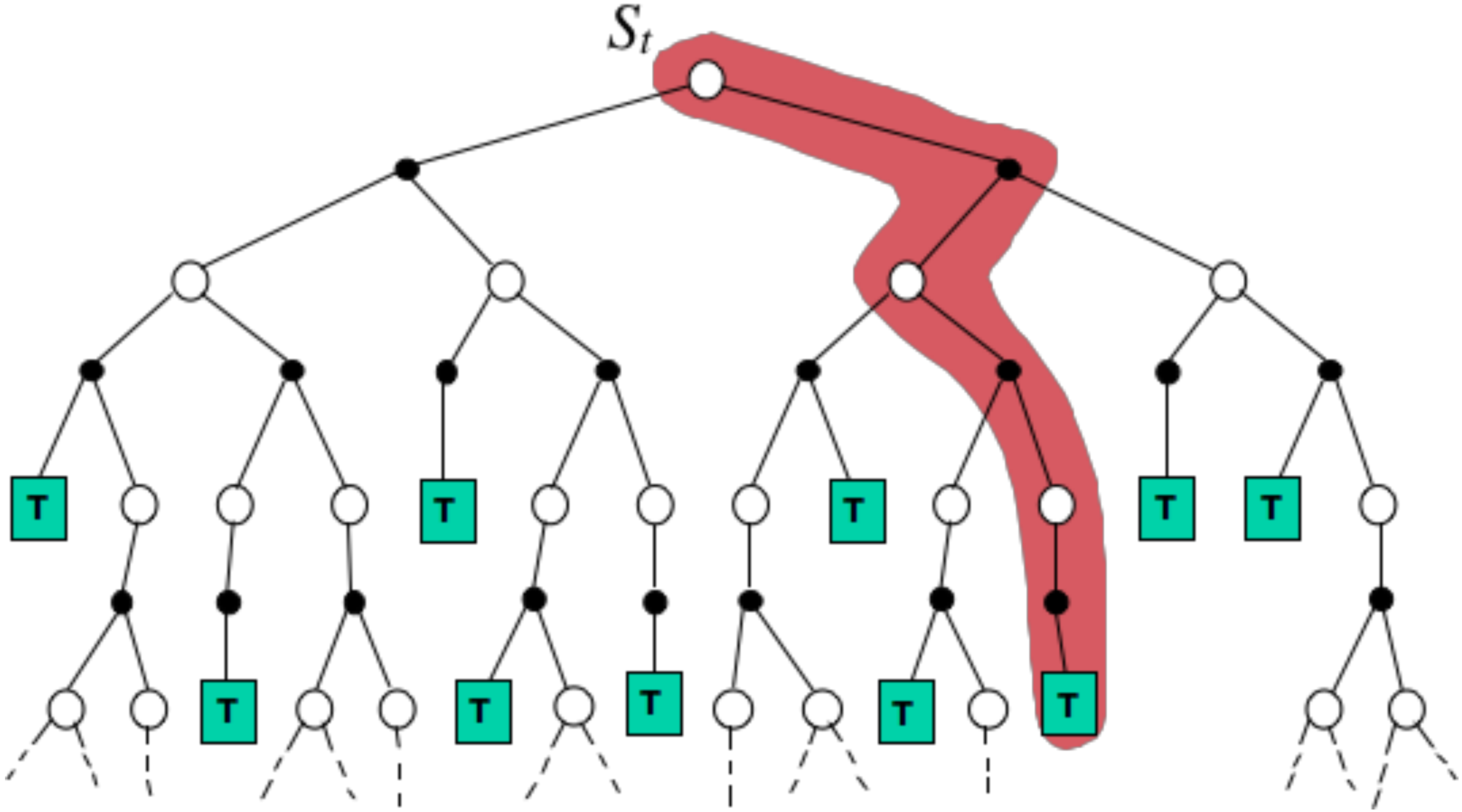
Dynamic Programming

$$V(S_t) \leftarrow E_\pi\Big[R_{t+1} + \gamma V(S_{t+1})\Big] = \sum_a \pi(a|S_t) \sum_{s',r} p(s',r|S_t,a)[r + \gamma V(s')]$$



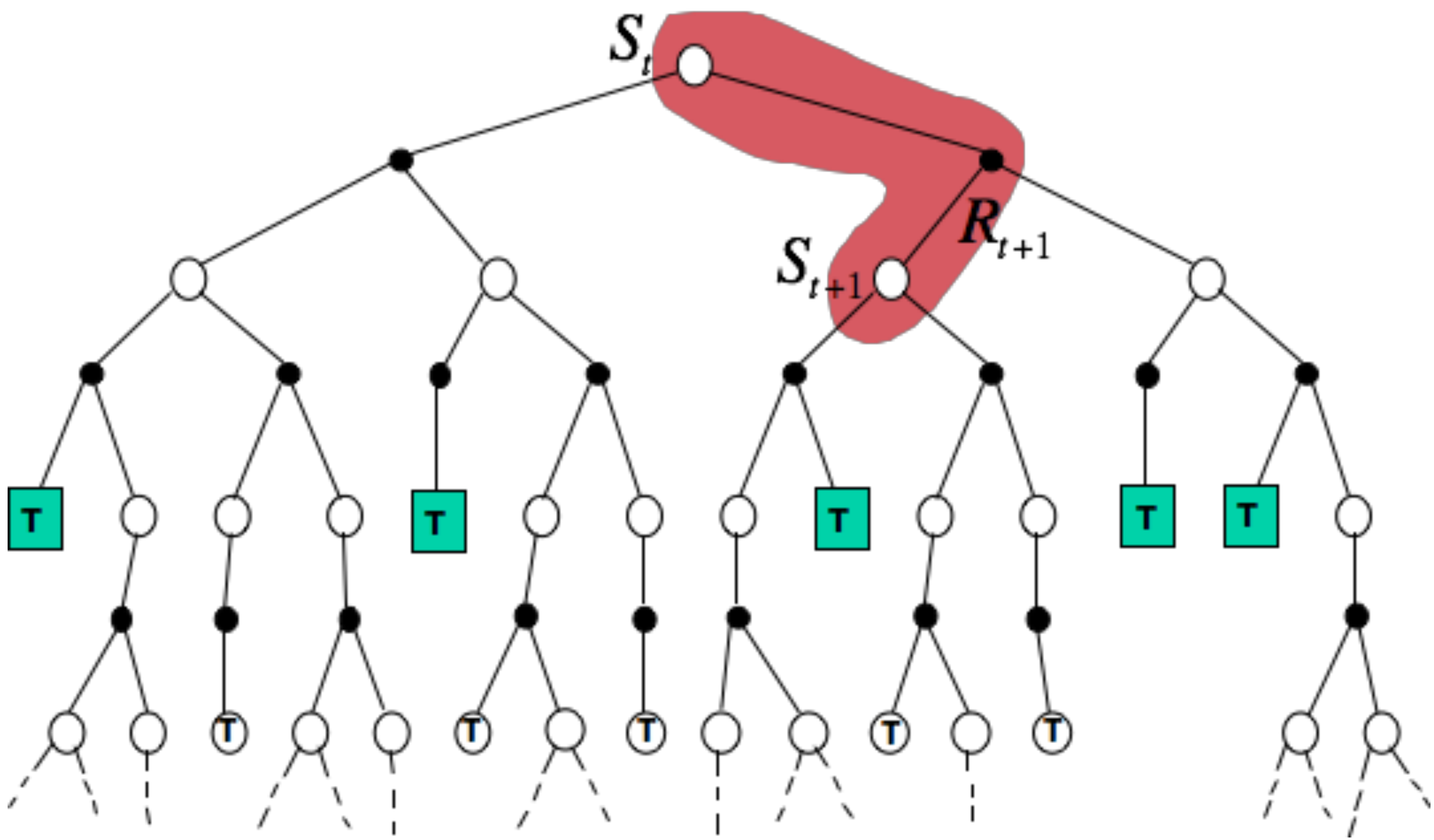
Monte Carlo

$$V(S_t) \leftarrow V(S_t) + \alpha \left(G_t - V(S_t) \right)$$



Simplest TD(0) Method

$$V(S_t) \leftarrow V(S_t) + \alpha \left(R_{t+1} + \gamma V(S_{t+1}) - V(S_t) \right)$$



TD Methods Bootstrap and Sample

- Bootstrapping: update involves an estimate
  - MC does not bootstrap
  - DP bootstraps
  - TD bootstraps
- Sampling: update does not involve an expected value
  - MC samples
  - DP does not sample
  - TD samples

TD Prediction

- Policy Evaluation (the prediction problem):
  - for a given policy π, compute the state-value function v_π

- Remember: Simple every-visit Monte Carlo method:

$$V(S_t) \leftarrow V(S_t) + \alpha\Big[G_t - V(S_t)\Big]$$

target: the actual return after time *t*

- The simplest Temporal-Difference method TD(0):

$$V(S_t) \leftarrow V(S_t) + \alpha\Big[\underbrace{R_{t+1} + \gamma V(S_{t+1})}_{\text{target}} - V(S_t)\Big]$$

target: an estimate of the return

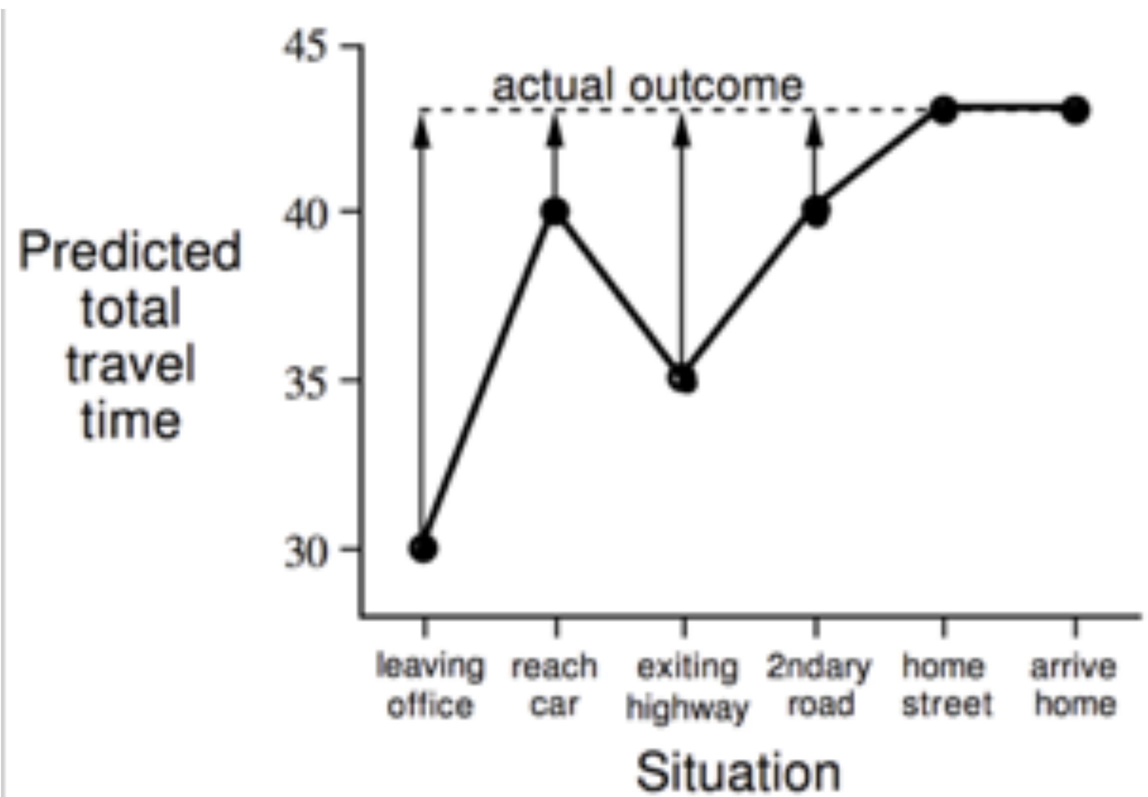
Example: Driving Home

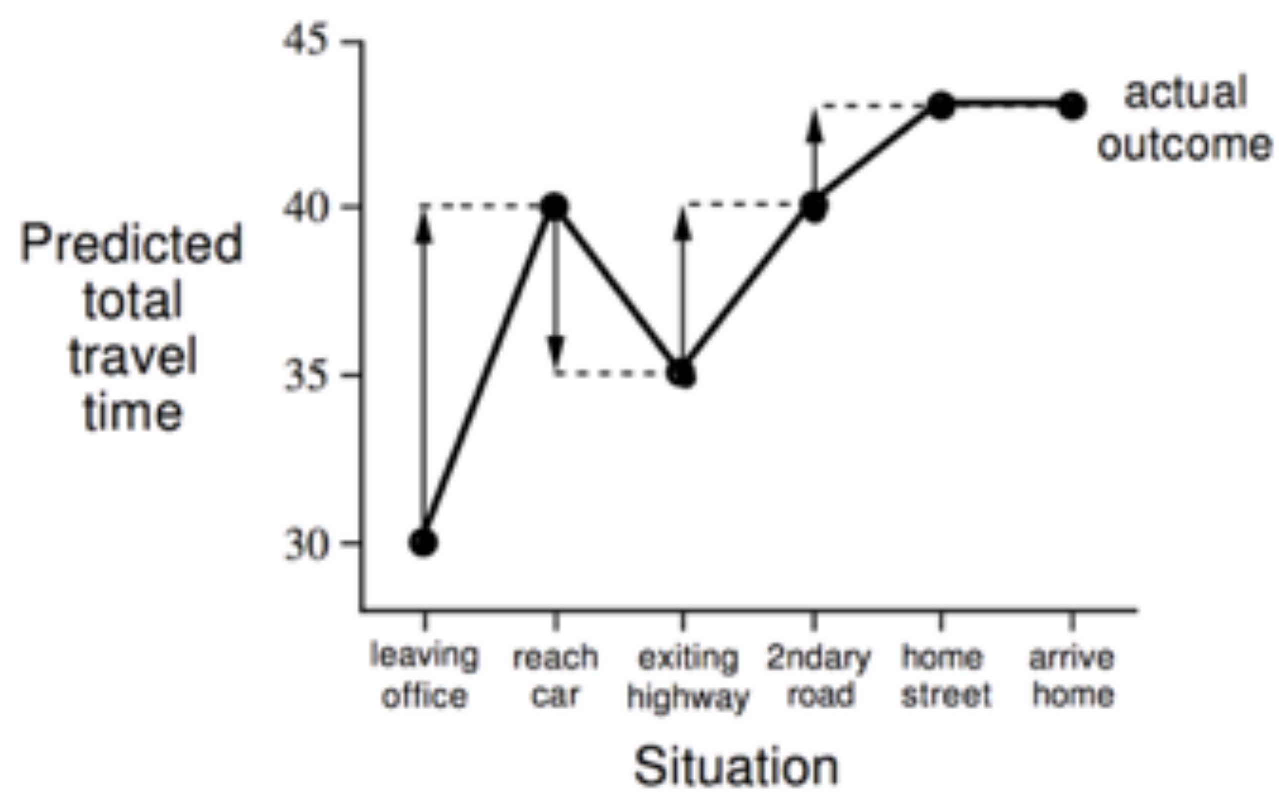
State	*Elapsed Time (minutes)*	*Predicted Time to Go*	*Predicted Total Time*
leaving office, friday at 6	0	30	30
reach car, raining	5	35	40
exiting highway	20	15	35
2ndary road, behind truck	30	10	40
entering home street	40	3	43
arrive home	43	0	43

Example: Driving Home

Changes recommended by Monte Carlo methods (α=1)

Changes recommended by TD methods (α=1)





Advantages of TD Learning

- TD methods do not require a model of the environment, only experience
- TD, but not MC, methods can be fully incremental
- You can learn before knowing the final outcome
  - Less memory
  - Less computation
- You can learn without the final outcome
  - From incomplete sequences
- Both MC and TD converge (under certain assumptions to be detailed later), but which is faster?

Batch Updating in TD and MC methods

- Batch Updating: train completely on a finite amount of data,
  - e.g., train repeatedly on 10 episodes until convergence.
- Compute updates according to TD or MC, but only update estimates after each complete pass through the data.
- For any finite Markov prediction task, under batch updating, TD converges for sufficiently small α.
- Constant-α MC also converges under these conditions, but may converge to a different answer.

AB Example

- Suppose you observe the following 8 episodes:

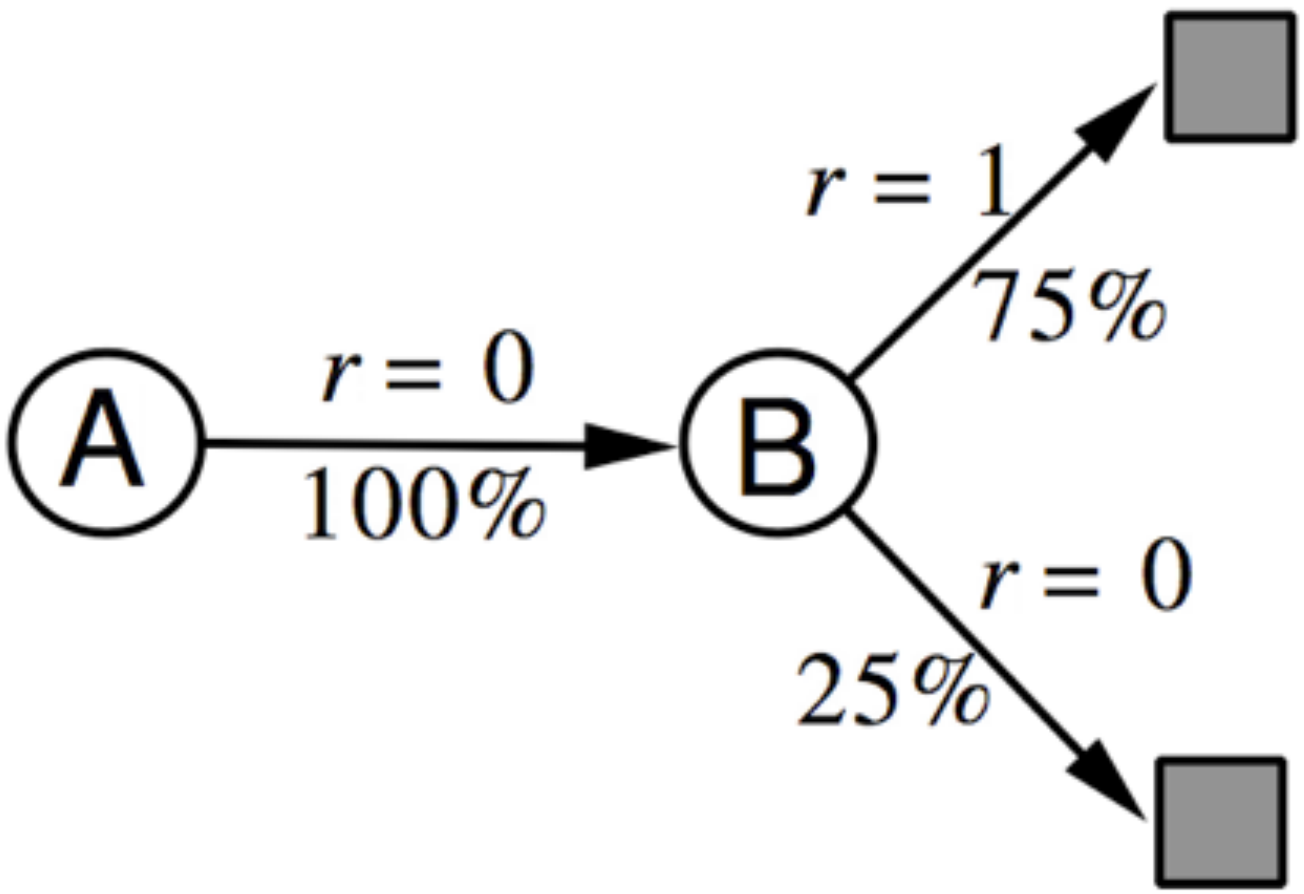
$$\begin{array}{l} A, 0, B, 0 \\ B, 1 \\ B, 1 \\ B, 1 \\ B, 1 \\ B, 1 \\ B, 1 \\ B, 0 \end{array}$$

$V(\mathrm{B})$? 0.75

$V(\mathrm{A})$? 0?

- Assume Markov states, no discounting (γ = 1)

AB Example



$V(\mathrm{A})$? 0.75

AB Example

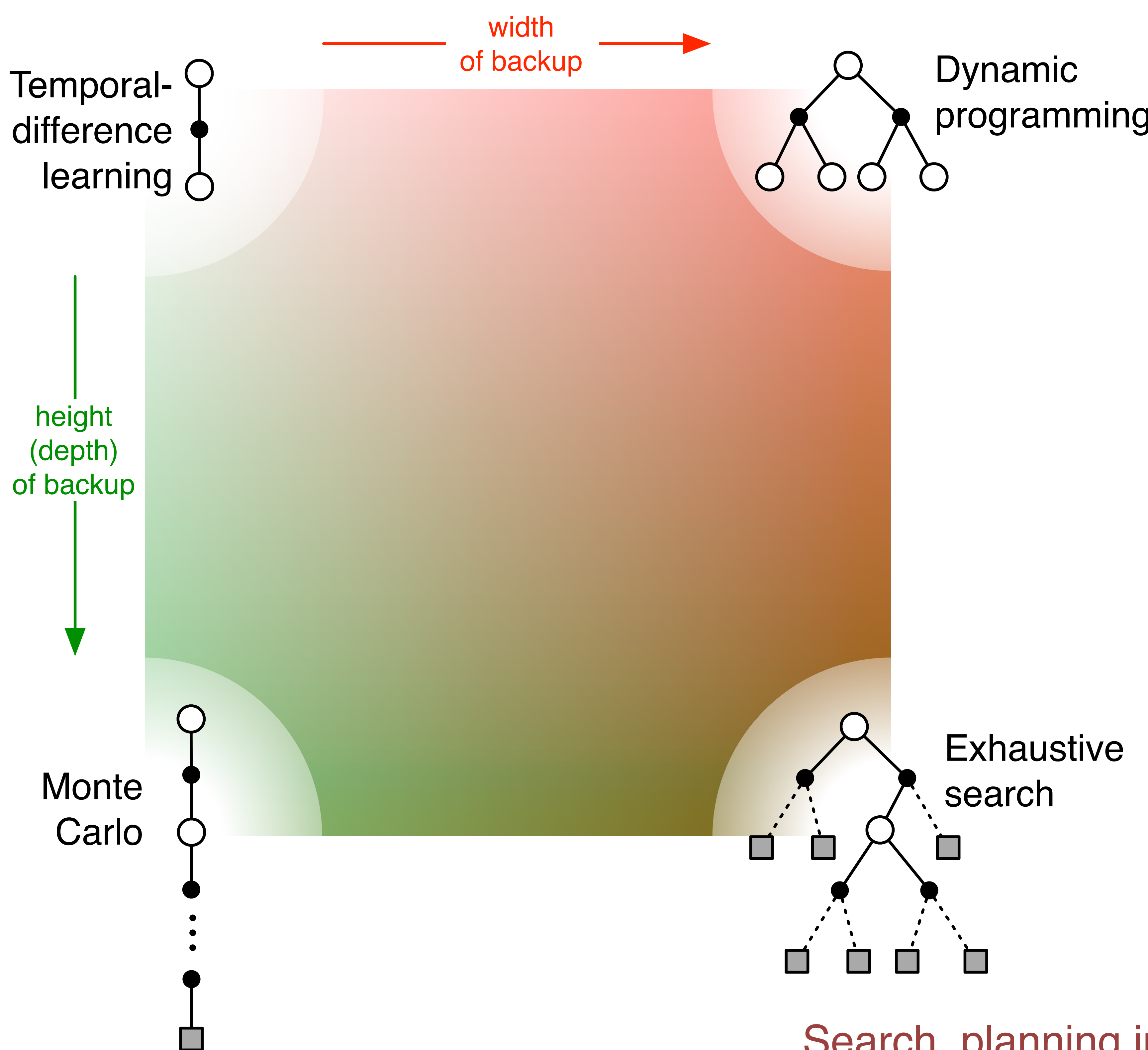
- The prediction that best matches the training data is V(A)=0
  - This minimizes the mean-square-error on the training set
  - This is what a batch Monte Carlo method gets

- If we consider the sequentiality of the problem, then we would set V(A)=.75
  - This is correct for the maximum likelihood estimate of a Markov model generating the data
  - i.e, if we do a best fit Markov model, and assume it is exactly correct, and then compute what it predicts.
  - This is called the certainty-equivalence estimate
  - This is what TD gets

Summary so far

- Introduced one-step tabular model-free TD methods
- These methods bootstrap and sample, combining aspects of DP and MC methods
- If the world is truly Markov, then TD methods will learn faster than MC methods

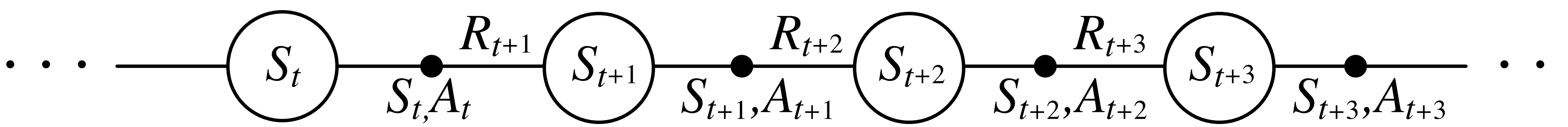
Unified View



Search, planning in a later lecture!

Learning An Action-Value Function

- Estimate q_π for the current policy π



After every transition from a nonterminal state, S_t, do this:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha\left[R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t)\right]$$

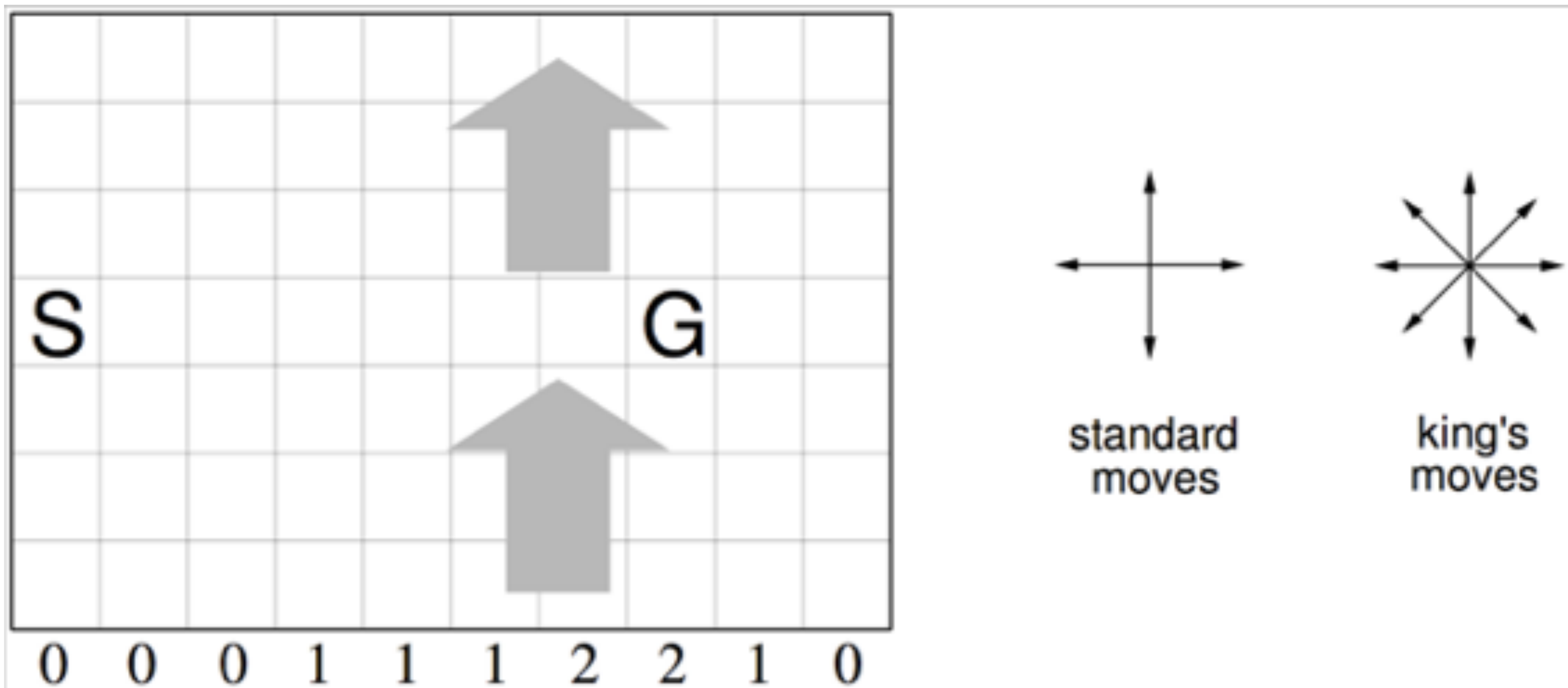
If S_{t+1} is terminal, then define $Q(S_{t+1}, A_{t+1}) = 0$

Sarsa: On-Policy TD Control

- Turn this into a control method by always updating the policy to be greedy with respect to the current estimate:

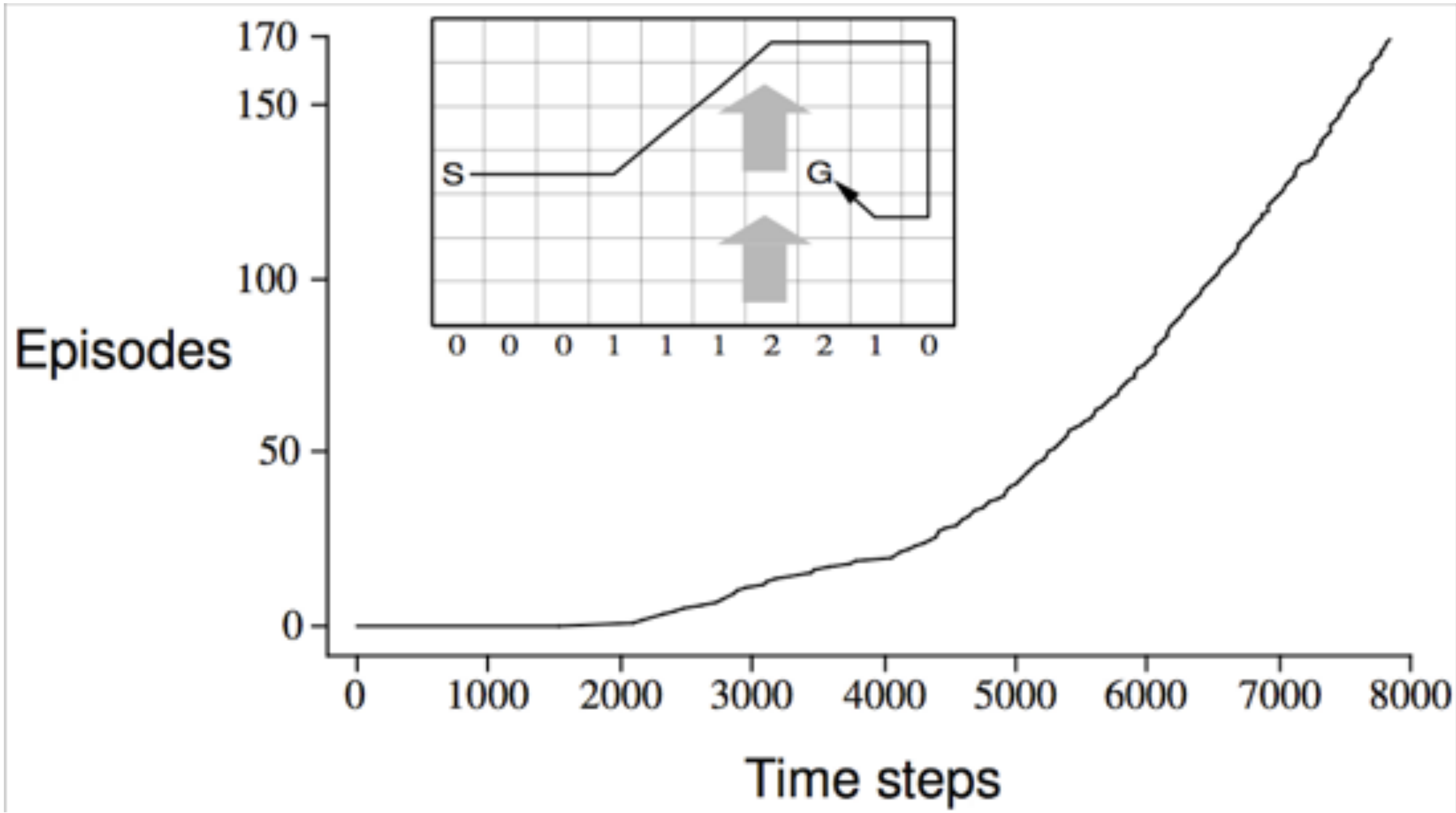
Initialize $Q(s,a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
  Initialize S
  Choose A from S using policy derived from Q (e.g., ε-greedy)
  Repeat (for each step of episode):
    Take action A, observe R, S'
    Choose A' from S' using policy derived from Q (e.g., ε-greedy)
    $Q(S,A) \leftarrow Q(S,A) + \alpha[R + \gamma Q(S',A') - Q(S,A)]$
    $S \leftarrow S'$; $A \leftarrow A'$;
  until S is terminal

Windy Gridworld



- undiscounted, episodic, reward = –1 until goal

Results of Sarsa on the Windy Gridworld



Q: Can a policy result in infinite loops? What will MC policy iteration do then?

- If the policy leads to infinite loop states, MC control will get trapped as the episode will not terminate.
- Instead, TD control can update continually the state-action values and switch to a different policy.

Q-Learning: Off-Policy TD Control

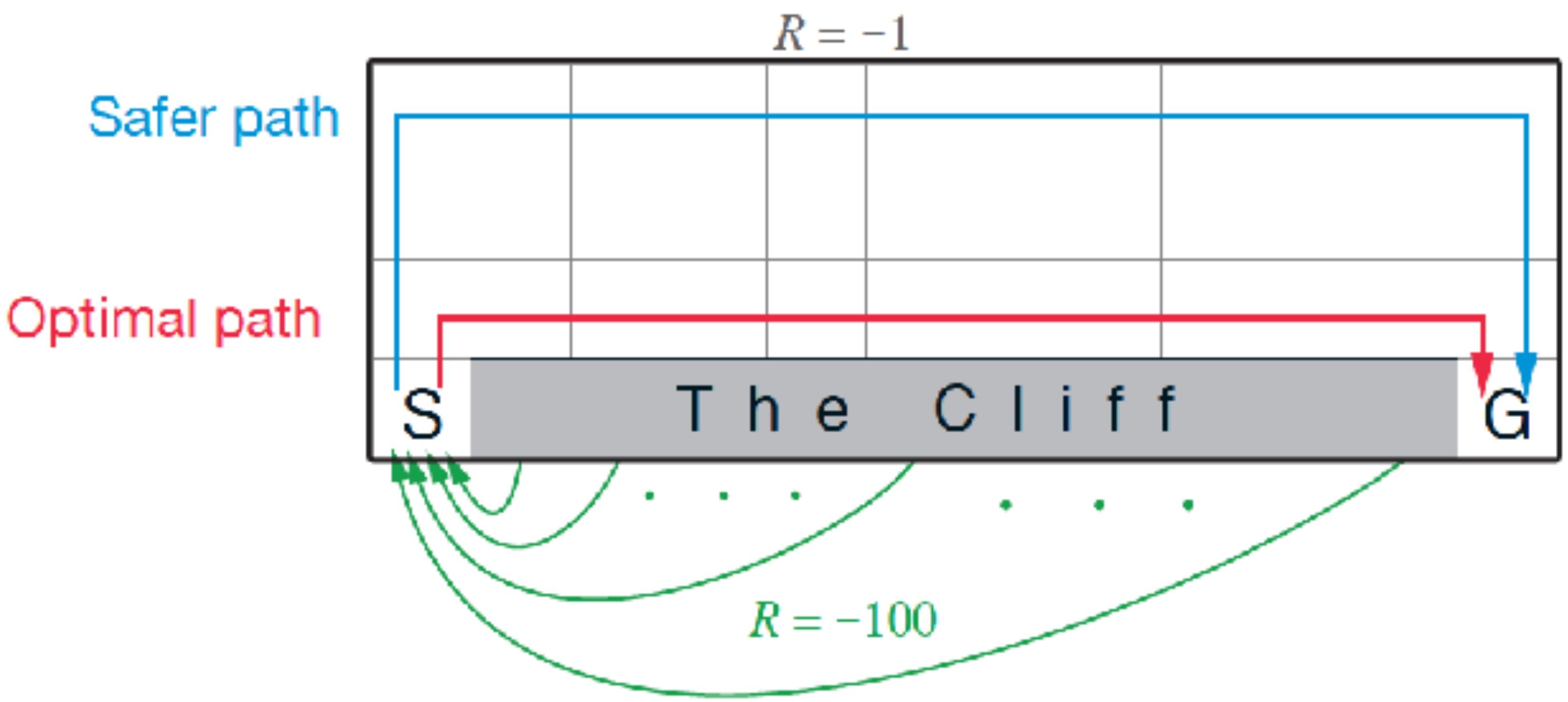
- One-step Q-learning:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha\Big[R_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t)\Big]$$

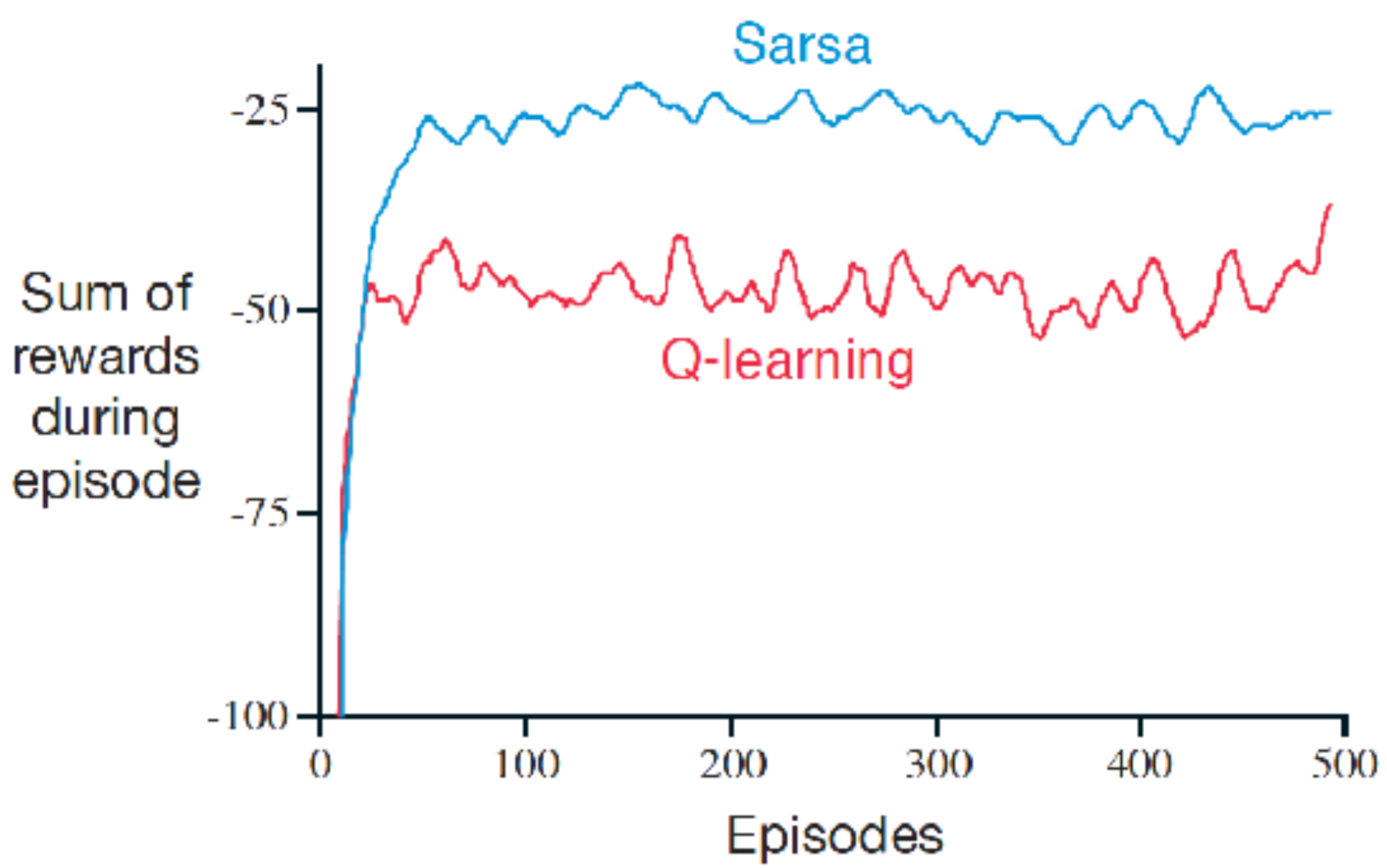
Initialize $Q(s,a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
    Initialize S
    Repeat (for each step of episode):
        Choose A from S using policy derived from Q (e.g., ε-greedy)
        Take action A, observe R, S'
        $Q(S,A) \leftarrow Q(S,A) + \alpha[R + \gamma \max_a Q(S',a) - Q(S,A)]$
        $S \leftarrow S'$;
    until S is terminal

$$Q(S,A) \leftarrow Q(S,A) + \alpha[R + \gamma Q(S',A') - Q(S,A)]$$

Cliffwalking



ϵ − greedy, $\epsilon = 0.1$



Expected Sarsa

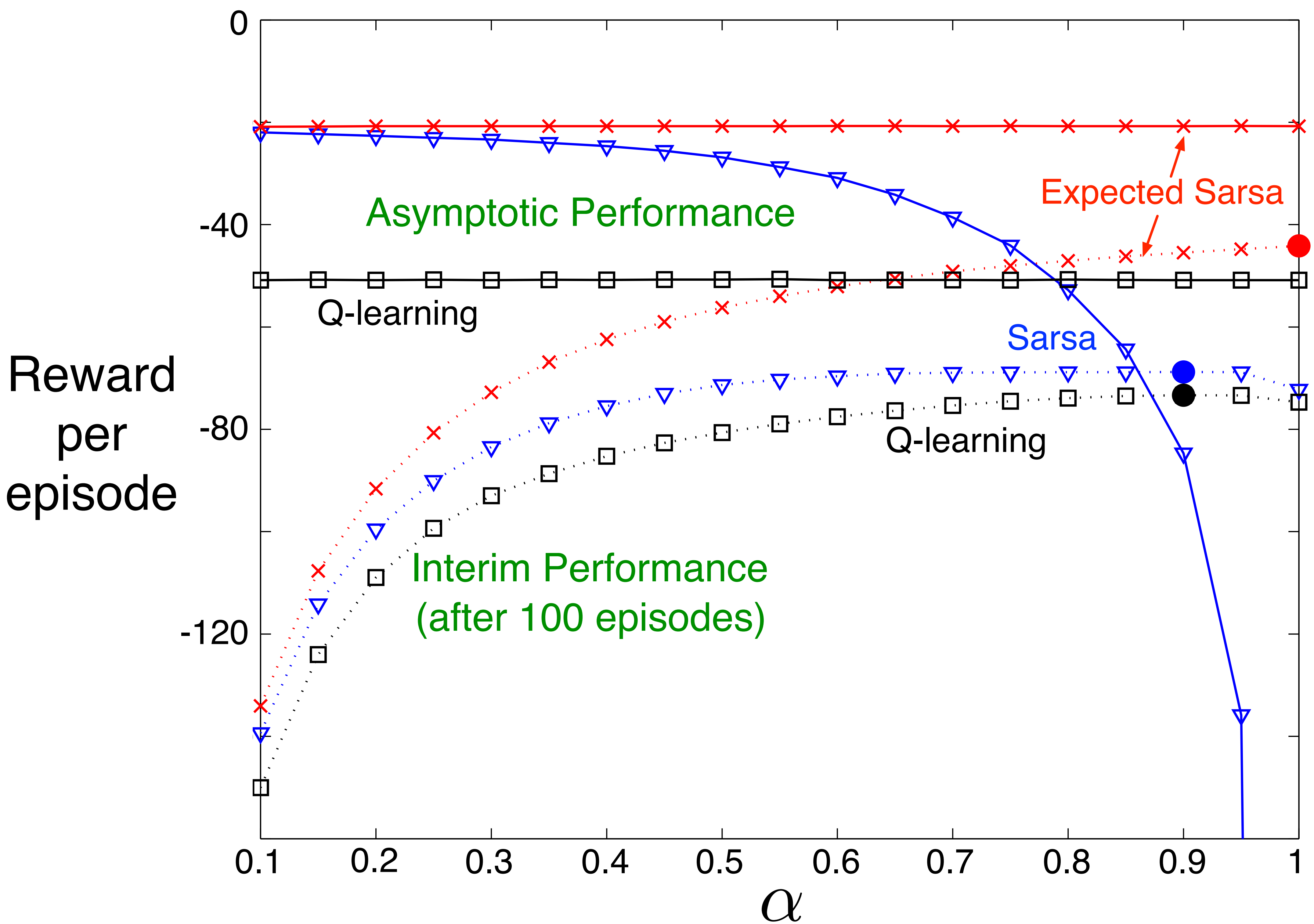
- Instead of the sample value-of-next-state, use the expectation!

$$
\begin{aligned}
Q(S_t, A_t) &\leftarrow Q(S_t, A_t) + \alpha\Big[R_{t+1} + \gamma \mathbb{E}[Q(S_{t+1}, A_{t+1}) \mid S_{t+1}] - Q(S_t, A_t)\Big] \\
&\leftarrow Q(S_t, A_t) + \alpha\Big[R_{t+1} + \gamma \sum \pi(a|S_{t+1}) Q(S_{t+1}, a) - Q(S_t, A_t)\Big]
\end{aligned}
$$

- Expected Sarsa performs better than Sarsa (but costs more)
  - Q: why?

Q: Is expected SARSA on policy or off policy? What if \pi is the greedy deterministic policy?

Performance on the Cliff-walking Task



Summary

- Introduced one-step tabular model-free TD methods
- These methods bootstrap and sample, combining aspects of DP and MC methods
- TD methods are computationally congenial
- If the world is truly Markov, then TD methods will learn faster than MC methods
- Extend prediction to control by employing some form of GPI
  - On-policy control: Sarsa, Expected Sarsa
  - Off-policy control: Q-learning