Planar Kinematics & Dynamics

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Recall that in the $x-y$ plane, all rotations are around the $+z$ axis. All kinematic quantities can be found by simply dropping the rows and columns corresponding to z translation as well as x and y rotations from the 3D version. (Note that in this table the R and g for 3D still assume only planar motion, just written in 3D).

Planar (2D)	Spatial (3D)
$R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$	$R_z = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$
$g = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & p_x \\ \sin(\theta) & \cos(\theta) & p_y \\ 0 & 0 & 1 \end{bmatrix}$	$g = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & p_x \\ \sin(\theta) & \cos(\theta) & 0 & p_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
$v = \begin{bmatrix} v_x \\ v_y \end{bmatrix}$	$v = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$
$w = [w_z]$	$w = \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix}$
$V = \begin{bmatrix} v_x \\ v_y \\ w_z \end{bmatrix}$	$V = \begin{bmatrix} v_x \\ v_y \\ v_z \\ w_x \\ w_y \\ w_z \end{bmatrix}$
$\hat{w} = \begin{bmatrix} 0 & -w_z \\ w_z & 0 \end{bmatrix}$	$\hat{w} = \begin{bmatrix} 0 & -w_z & w_y \\ w_z & 0 & -w_x \\ -w_y & w_x & 0 \end{bmatrix}$
$\hat{V} = \begin{bmatrix} 0 & -w_z & v_x \\ w_z & 0 & v_y \\ 0 & 0 & 0 \end{bmatrix}$	$\hat{V} = \begin{bmatrix} 0 & -w_z & w_y & v_x \\ w_z & 0 & -w_x & v_y \\ -w_y & w_x & 0 & v_z \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Planar (2D)	Spatial (3D)
	$Ad_g = \begin{bmatrix} R & \hat{p}R \\ 0 & R \end{bmatrix}$
$Ad_g = \begin{bmatrix} c & -s & p_y \\ s & c & -p_x \\ 0 & 0 & 1 \end{bmatrix}$	$= \begin{bmatrix} c & -s & 0 & 0 & 0 & p_y \\ s & c & 0 & 0 & 0 & -p_x \\ 0 & 0 & 1 & -p_y c + p_x s & p_y s + p_x c & 0 \\ 0 & 0 & 0 & c & -s & 0 \\ 0 & 0 & 0 & s & c & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$
for: $c = \cos(\theta), \quad s = \sin(\theta)$	Note: $Ad_g V$ for planar V means columns 3, 4, and 5 are irrelevant (will multiply a 0 in V)
$F = \begin{bmatrix} f_x \\ f_y \\ \tau_z \end{bmatrix}$	$F = \begin{bmatrix} f_x \\ f_y \\ f_z \\ \tau_x \\ \tau_y \\ \tau_z \end{bmatrix}$
For frictional point contact:	
$f_c = \begin{bmatrix} f_{cx} \\ f_{cy} \end{bmatrix}$	$f_c = \begin{bmatrix} f_{cx} \\ f_{cy} \\ f_{cz} \end{bmatrix}$
$B_c = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$	$B_c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
$M_o = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_z \end{bmatrix}$	$M_o = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_x & 0 & 0 \\ 0 & 0 & 0 & 0 & I_y & 0 \\ 0 & 0 & 0 & 0 & 0 & I_z \end{bmatrix}$
	For reference frame aligned with major axes.