Aggregating Strategic Information

James Best[1] Maryam Saeedi[1]
Ali Shourideh[1] Daniel Quigley[2]

[1]Department of Economics, Tepper School of Business, CMU

[2]Department of Economics, University of Oxford

June 2024
Cowles Annual Economic Theory Conference 2024

Aggregating Information

- Decision-makers often rely on information from multiple sources
  1. Board relies on reports of multiple divisions
  2. CB rate decisions communicate views of many board members
  3. Online trade and review aggregation: Yelp, IMDb, Goodreads

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- Common issue: conflicts of interest & strategic manipulation

This Paper

- Is there a way for an aggregator to overcome this strategic manipulation while being informative?
- What are the properties of optimal mechanism?
- How does this mechanism depend on
  - the level of “conflict”
  - the number of senders

Literature

Multi-sender/issue cheap talk: Austen-Smith (1993), Krishna & Morgan (2001), Battaglini (2002, 2004), Ambrus et al. (2013), Meyer et al. (2019), Lipnowski and Ravid (2020), Antic et al (2023).

Mediation/mechanism design in communication games: Wolinsky (2002), Krishna & Morgan (2008), Goltsman et al. (2009), Salamanca (2020), Jann & Schottmüller (2023).

Mechanism Design without transfers: Börgers & Postl (2009), Gershkov et al. (2017), Li et al (2017), Guo & Hörner (2018), Kattwinkel, et al (2022), Kattwinkel & Winter (in progress)

Incentives in information design: Onuchic and Ray (2022), Boleslavski and Kim (2023), Saeedi and Shourideh (2023)

This paper:
Optimal multi-sender communication mechanisms without transfers or cross-checking.

Roadmap

Model

Results

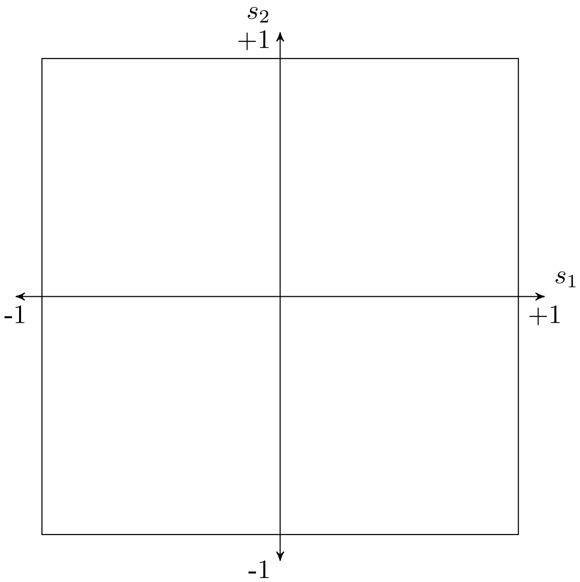
Simple & Approximate Implementation

Delegation Problem

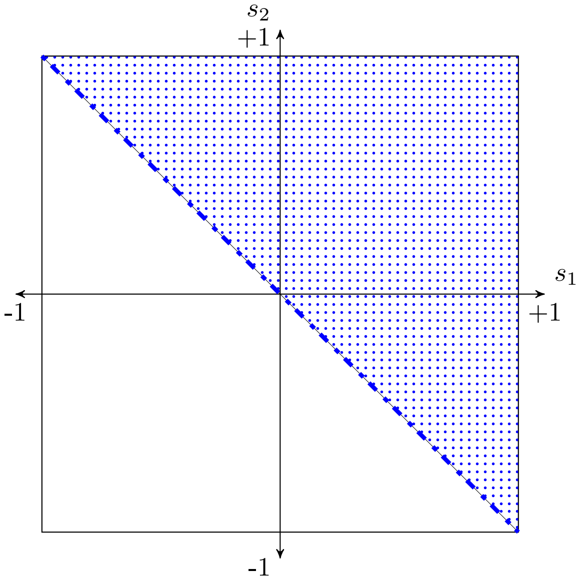
Model

- n **senders** each with **iid** type $s_i \sim F[-1,1]$,
  - where $\mathbf{s} = (s_i)_{i=1}^n$.
- A **receiver** with binary action $a \in \{0,1\}$.
- Payoff relevant **state variable** $\omega = \frac{\sum_{i=1}^n s_i}{n}$
- Receiver Payoff: $a\omega$.
- Senders' Payoff: $a(\omega + b)$ with $b > 0$.
  - Biased toward $a = 1$

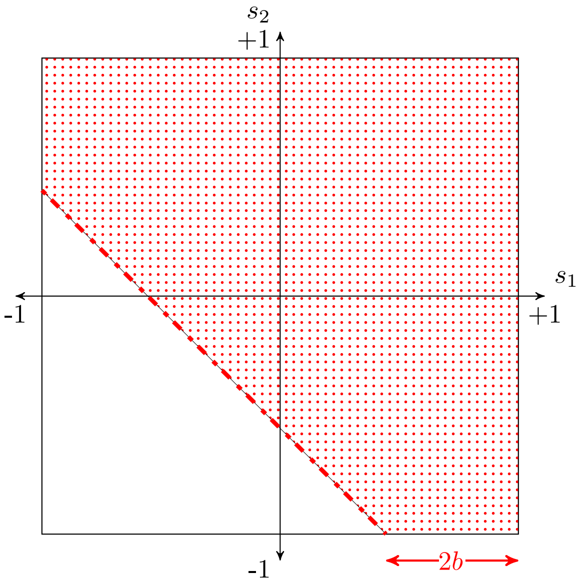
Two Sender Case



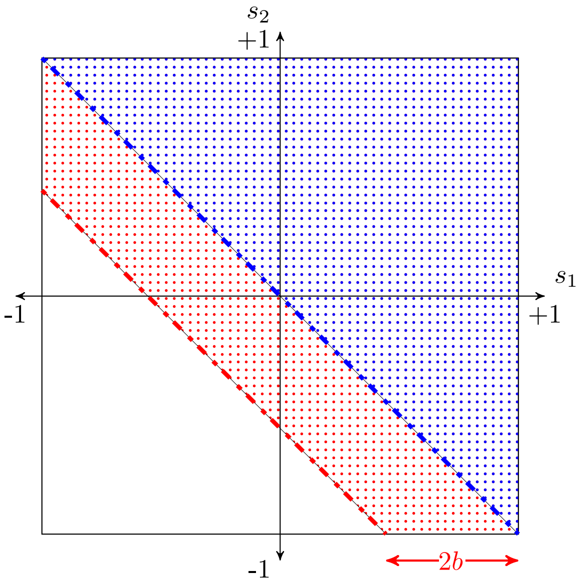
Two Sender Case, Receiver Optimal Action



Two Sender Case, Senders Optimal Action



Disagreement Region



Mediator with Commitment

- **Mediator** with commitment, e.g., a review aggregator
- Commits to a mechanism $\sigma : [-1, 1]^n \to \Delta\{0, 1\}$.
  - Myerson (1982, 1986): WLOG, direct mechanisms
- Each sender reports type $s_i \in [-1, 1]$ to the mediator
- The mediator recommends action $\tilde{a} = 1$ with probability equal to $\sigma(\mathbf{s} = (s_i)_{i=1}^n)$.
- After observing $\tilde{a}$, receiver chooses $a \in \{0, 1\}$ and payoffs are realized.

Mediator Problem, Maximizing Receiver Payoff

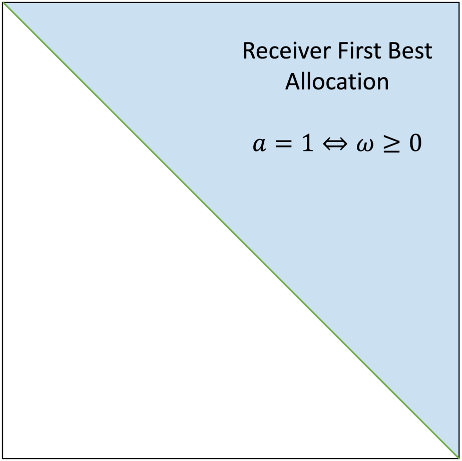
$$\max_{\sigma} \mathbb{E}\left[\sigma(\mathbf{s})\omega\right]$$

Subject to

$$s_i \in \arg\max_{\tilde{s}} \mathbb{E}\left[\sigma(\tilde{s}, \mathbf{s}_{-i})\left(\omega + b\right)\right] \tag{IC}$$

$$\mathbb{E}_{\sigma}\left[\omega \mid \tilde{a}\right]\left(2\tilde{a} - 1\right) \geq 0 \tag{OB}$$

Allocations and Incentives: Two Senders



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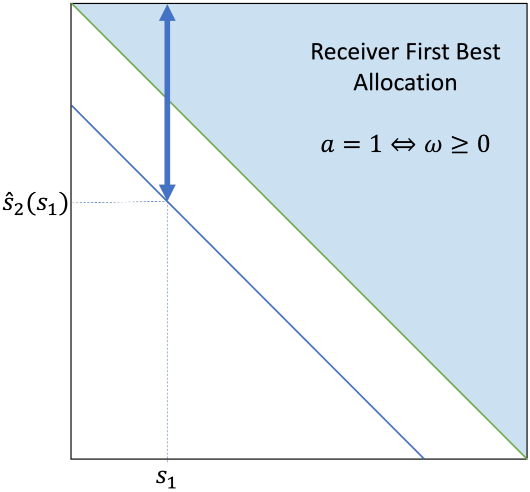

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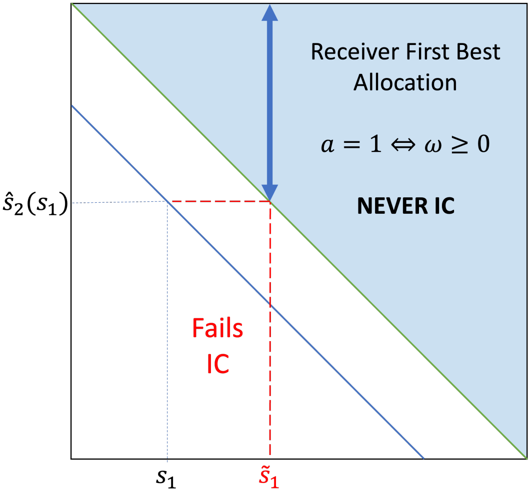

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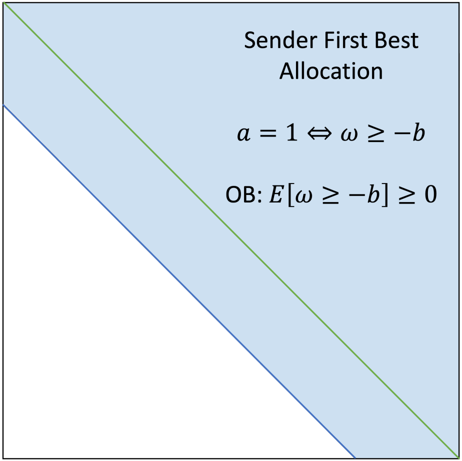

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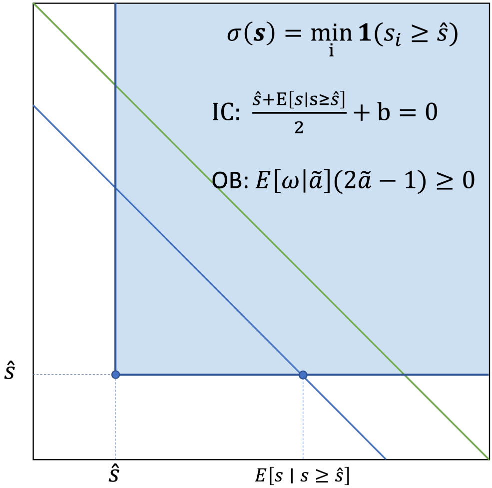

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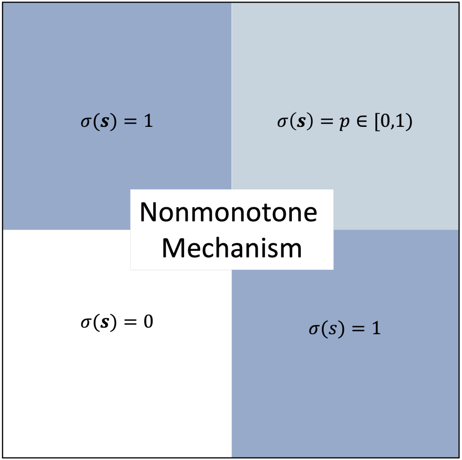

Definition

Roadmap

Small Bias: Sender Preferred Mechanism

Theorem. The optimal mechanism induces the senders' first best allocation if

$$1 - bn\left(1 - \frac{f'(x)}{f(x)}(1-x)\right) \geq 0,\ \forall x \in [-1, 1] \tag{1}$$

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- This condition only holds if $bn \leq 1$
- Sender first best is optimal when
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- f'/f imposes conditions on density (more on it later)

Overview of Proof

Two main steps:

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  - Standard methods do not work: non-empty interior in L^∞
  - Use Mitter (2008), perturbing incentive constraints are bounded by a linear function
    - As in Kleiner & Manelli (2019) and Kushnir & Shourideh (2024)

Overview of Proof

Two main steps:

- Strong duality holds
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  - Use Mitter (2008), perturbing incentive constraints are bounded by a linear function
    - As in Kleiner & Manelli (2019) and Kushnir & Shourideh (2024)
- Constructing Lagrange Multipliers and proving that using those the only solution is seller preferred allocation
  - After many steps of algebra, we get to a term that is in form of

$$\sigma(s)(\omega + b)A$$

  - The assumptions ensures that A is positive.
  - Then to maximize objective set σ
    - Equal to 1, if $(\omega + b) > 0$
    - Equal to 0, if $(\omega + b) < 0$

Small Bias: Uniform Distribution

Corollary. If the distributions of types are uniform, then the optimal mechanism induced by the senders' first best allocation if and only if

$$b \leq 1/n.$$

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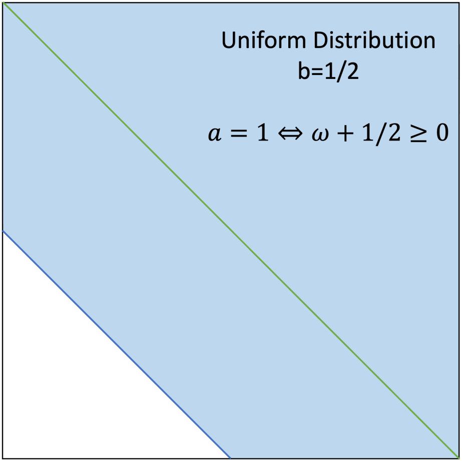
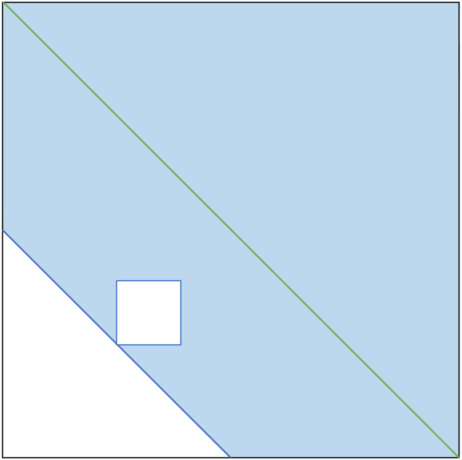
- The higher the number of senders, the smaller the bias that induces senders first best
- Each sender has smaller share of the total information

Intuition: Two Senders and Uniform Distribution



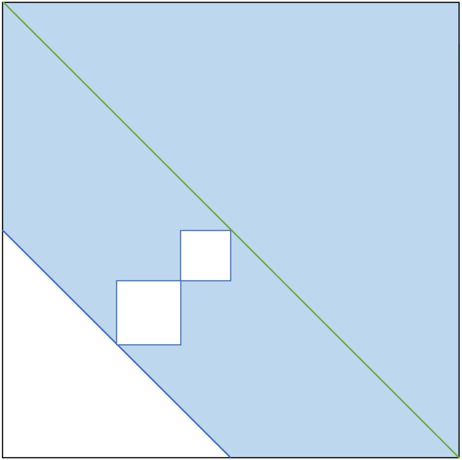
- What prevents us from setting $\sigma = 0$ in the disagreement region?

Intuition: Two Senders and Uniform Distribution



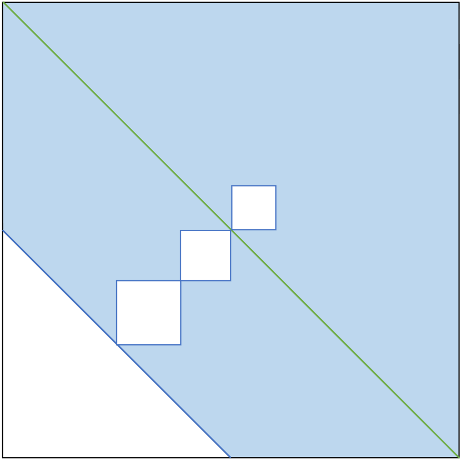
- Let's assume that we set $\sigma = 0$ in disagreement region as above.
- To make the IC for these types hold, should make higher types worse off.

Intuition: Two Senders and Uniform Distribution



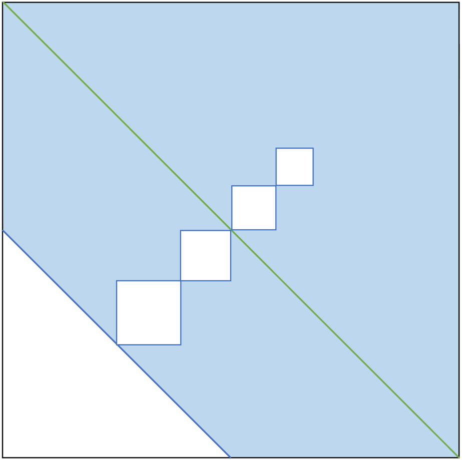
- Next step, more valuable ω, we can make the deviation smaller
- But we need to keep going!

Intuition: Two Senders and Uniform Distribution



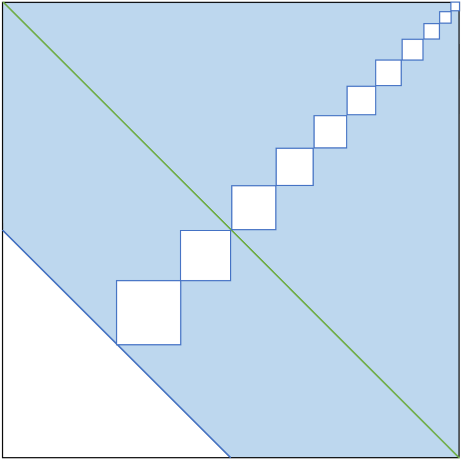
- We need to keep making higher levels of s worse off even in the agreement region.

Intuition: Two Senders and Uniform Distribution _



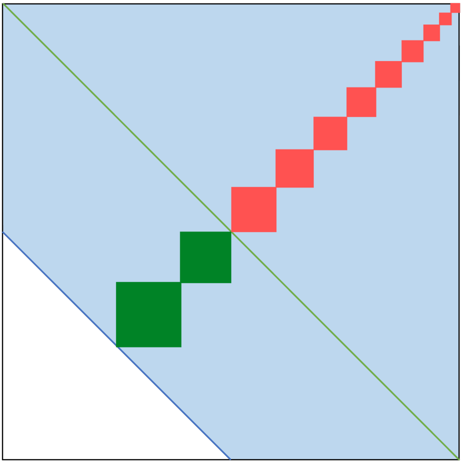
- Keep going until ...

Intuition: Two Senders and Uniform Distribution _



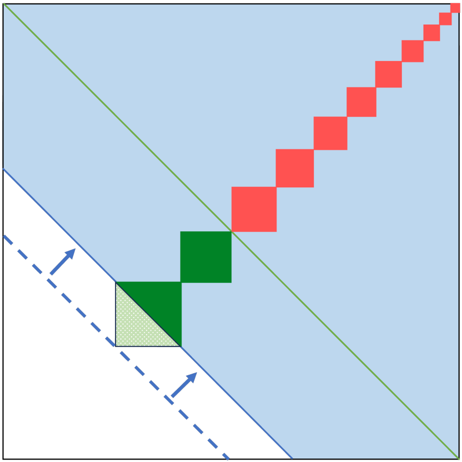
- We get to $s_1, s_2 = 1$

Intuition: Two Senders and Uniform Distribution _



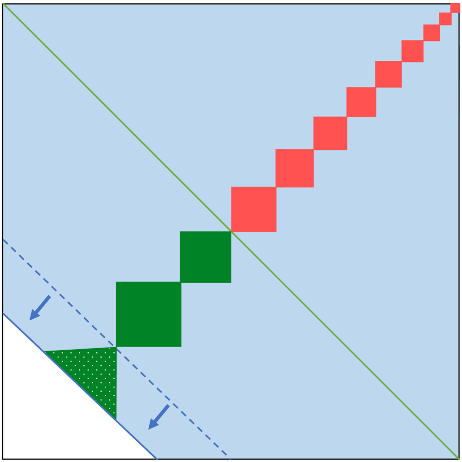
- For the receiver, in the case of uniform and $b = 1/2$
  - The benefit (green) = The cost (red)

Intuition: Two Senders and Uniform Distribution _



- For the receiver, in the case of uniform and $b < 1/2$
    - The benefit (green) < The cost (red)

Intuition: Two Senders and Uniform Distribution



- For the receiver, in the case of uniform and $b > 1/2$
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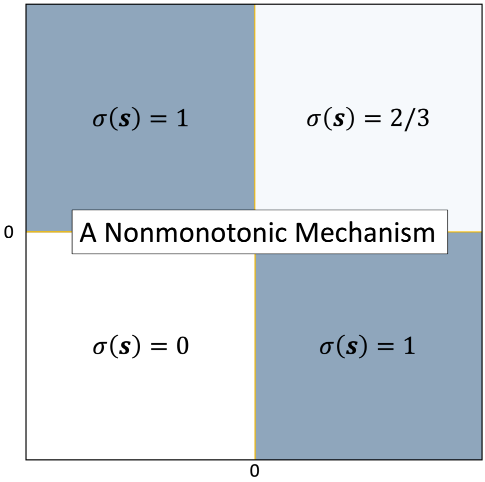
Intuition: Two Senders and Uniform Distribution



- At $b = 1/2$ above non-monotone mechanism satisfies IC
- Gives receiver the same payoff as sender best allocation

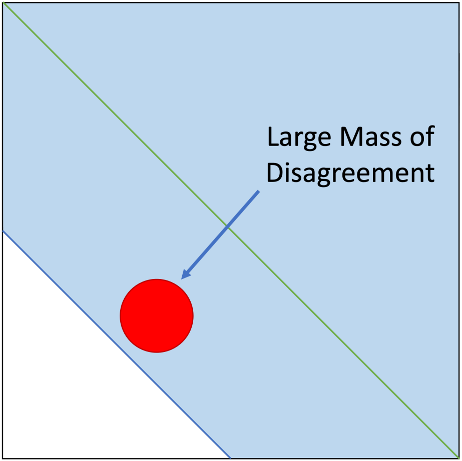
Condition on the distribution

- Condition (1):

$$1 - bn\left(1 - \frac{f'(x)}{f(x)}(1-x)\right) \geq 0$$

- Density cannot be declining too fast.
- Somewhat similar to increasing virtual values in Myerson (81).

Intuition: Condition (1)



- Our condition ensures that there is no big mass in the disagreement region

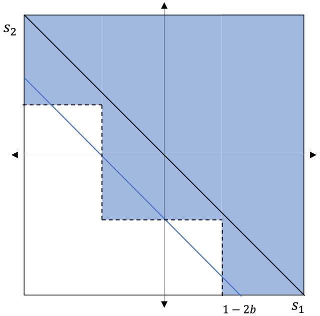
Large bias: Non-monotone Mechanism

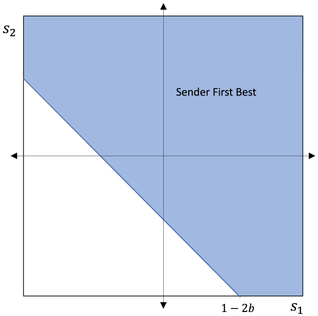
Proposition. There exists a $\bar{b} \in [\frac{1}{n}, 1)$ such that the optimal mechanism is non-monotonic for all $b > \bar{b}$.

Large bias: Non-monotone Mechanism

Proof Outline

- We first show that all IC and monotone mechanisms have one of the following two forms or their combinations.





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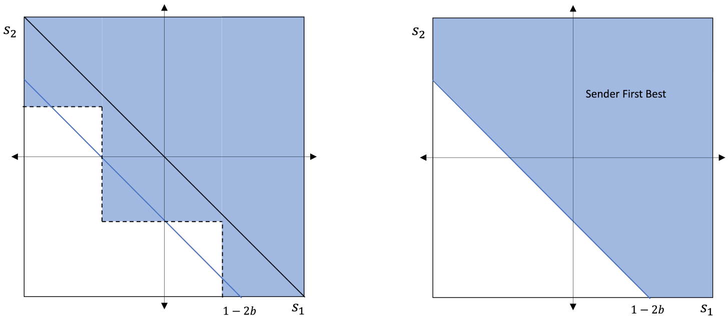
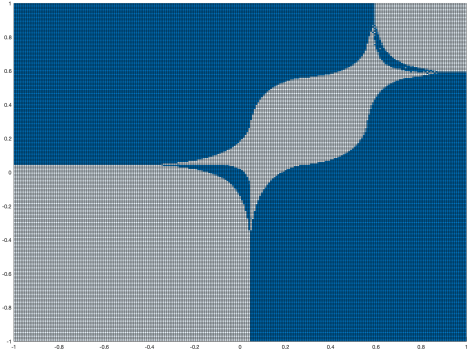


- Construct an upper bound on $\mathbb{E}_\sigma u^R$ from monotonic σ
- Show that it converges to zero as $b \to 1$.
- Show there always exists an informative non-monotonic mechanism.

Optimal Non-monotone Mechanisms!

$$b = .6, s \sim U\left[-1, 1\right]$$



- Blue area: $\sigma = 1$, the rest, $\sigma = 0$

Roadmap

Simple & Approximate Implementation

- Assume that the mediator restricts itself to a *simpler* mechanisms
- The mediator asks the senders if their signal was positive or negative (Thumbs up/Thumbs down)
- Then implements a mechanism as a function of number of positive signals: $\sigma(k)$
- This will greatly simplify IC's and the number of parameters for σ
- Let's assume that the distribution of signals are uniform for the presentation (Not necessary)

Simple & Approximate Implementation

- The problem of the mediator and IC will become maximizing the following:

$$V_n(\sigma(k)) = \sum_{k=0}^{n} \binom{n}{k} \sigma(k)\mathbb{E}(\omega|k) \tag{2}$$

- Subject to the IC for $s = 0$:

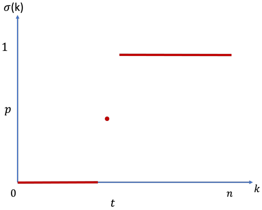
$$\sum_{k=0}^{n-1} \binom{n-1}{k} (\sigma(k+1) - \sigma(k))\mathbb{E}(\omega + b|s = 0, k) = 0$$

- Monotonicity and Obedience

Simple & Approximate Implementation

- We show that the optimal σ has one of the following two forms:

Monotone Mechanism



$$\sigma(k) = \begin{cases} 1 & k > t \\ p \in (0,1] & k = t \\ 0 & k < t \end{cases}$$

Simple & Approximate Implementation

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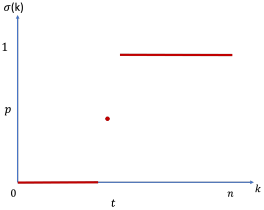
Monotone Mechanism



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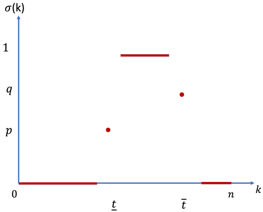
Non-monotone Mechanism



$$\sigma(k) = \begin{cases} 1 & \underline{t} < k < \bar{t} \\ p \in (0,1] & k = \underline{t} \\ q \in (0,1] & k = \bar{t} \\ 0 & \text{otherwise} \end{cases}$$

Simple & Approximate Implementation

- For low levels of b the monotone mechanism will be optimal
  - This is the closest approximation to sender best
- For high levels of b the non-monotone will be optimal
  - The switch happens exactly at $b = 1/n$
  - Similar to the non-monotone mechanism for two sender.

Roadmap

Model

Results

Simple & Approximate Implementation

Delegation Problem

Parallel to the Delegation Problem

- In our problem we assume that the mediator gets information from multiple sources
- What if the mediator, has its own private signal and wants to elicit only one other source
  - s_2 is directly observed
  - Elicit information from sender 1: s_1

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- In our problem we assume that the mediator gets information from multiple sources
- What if the mediator, has its own private signal and wants to elicit only one other source
  - s_2 is directly observed
  - Elicit information from sender 1: s_1
- The designer's problem is

$$\max \mathbb{E}[\frac{(s_1 + s_2)}{2}\sigma(s_1, s_2)]$$

Subject to

$$s_1 \in \arg\max_{\tilde{s}} \mathbb{E}\left[\sigma(\tilde{s}, s_2)\left(\frac{(s_1 + s_2)}{2} + b\right)\right] \tag{IC}$$

Parallel to the Delegation

Theorem. Assuming inverse hazard rate, $\frac{1-F}{f}$, is non-increasing, the following mechanism maximizes the receiver's expected payoff:

1. If $1 + \mathbb{E}[s] < 2b$:

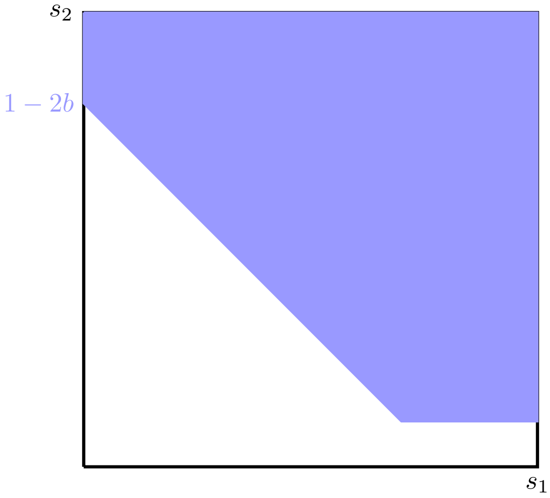
$$\sigma^\star(s_1, s_2) = \begin{cases} 1, & \text{if } s_2 \geq -\mathbb{E}[s_1] \\ 0, & \text{otherwise.} \end{cases}$$

2. If $1 + \mathbb{E}[s] \geq 2b$:

$$\sigma^\star(s_1, s_2) = \begin{cases} 1, & \text{if } s_2 \geq -\bar{s}_1 - 2b \text{ and } s_1 \geq -s_2 - 2b \\ 0, & \text{otherwise.} \end{cases}$$

   where $\bar{s}_1 \in (-1, 1)$ is the unique solution to $\mathbb{E}[s_1 \mid s_1 \geq s_1'] = s_1' + 2b$.

Parallel to the Delegation - Low Bias



Parallel to the Delegation

- High bias:
  - Ignore the signal from sender 1
- Low bias:
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- High bias:
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- Low bias:
  - If own signal is too low: recommend action 0
  - Otherwise, privately let sender 1 know your signal and delegate
- In this case, we do not have a non-monotone mechanism.
- Non-monotonicity is a feature of incentivizing multiple senders at the same time
- One sender case: same as interval delegation:
  - Amador, Werning, and Angeletos (2006), Alonso and Matouschek (2008), Amador and Bagwell (2013), Halac and Yared (2014, 2018, 2020, 2022, ...?), ...

Takeways

- When the bias and number of senders are small:
  - Sender best is optimal
  - Mediator can implement this by allowing the senders to talk freely and propose the action to reciever
- High bias or large number of sender
  - The mediator can improve the outcome for the receiver by implementing a non-monotone mechanism
  - Amazon retracting high ratings from time to time

Thank you!

Monotonic versus Nonmonotonic Mechanisms

Back

We say a mechanism is (ex post) monotonic IFF

$$\sigma(\tilde{s}, \mathbf{s}_{-i}) - \sigma(s_i, \mathbf{s}_{-i}) \geq 0 \text{ for all } \tilde{s} \geq s_i,\ \mathbf{s}_{-i} \in [-1, 1]^{n-1}$$

Monotonic versus Nonmonotonic Mechanisms Back

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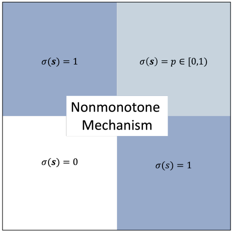

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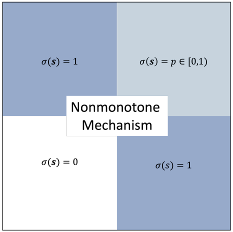

This is distinct from interim monotonicity, which is always required by IC:

$$\mathbb{E}[\sigma(\tilde{s}, \mathbf{s}_{-i})] \text{ is non-decreasing in } \tilde{s}.$$