When can price discrimination benefit consumers?

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A classic question: Pigou, Robinson, Varian, ...

Is price discrimination bad? (for CS, TS, ...)

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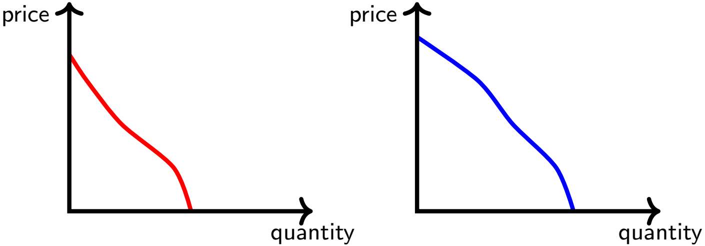

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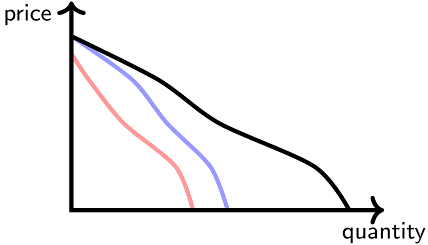

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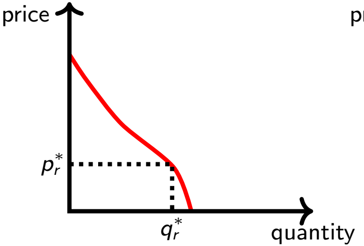

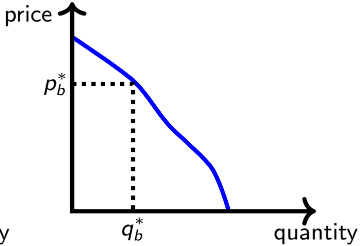

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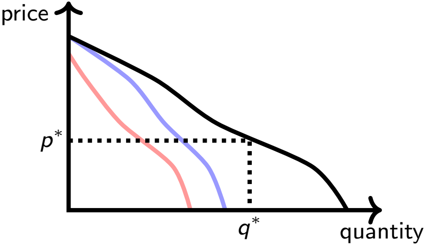

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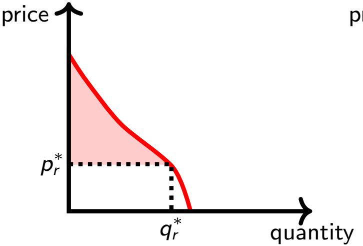

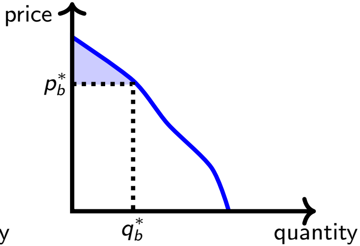

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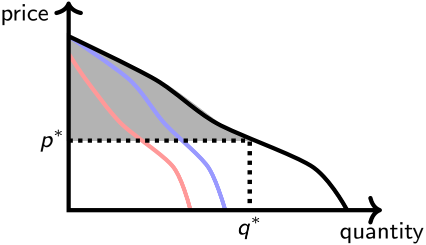

Pigou: "Yes! (with linear demands)"

Is price discrimination bad? (for CS, TS, ...)

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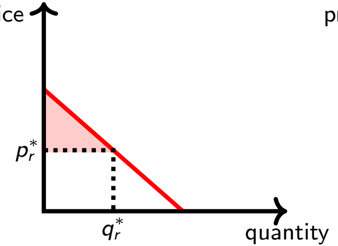

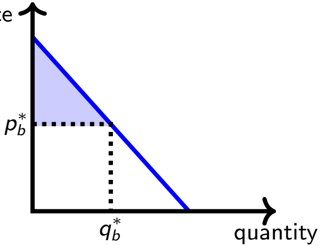

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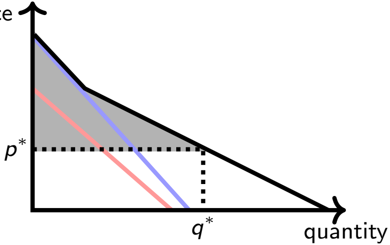

This paper: consider all segmentations

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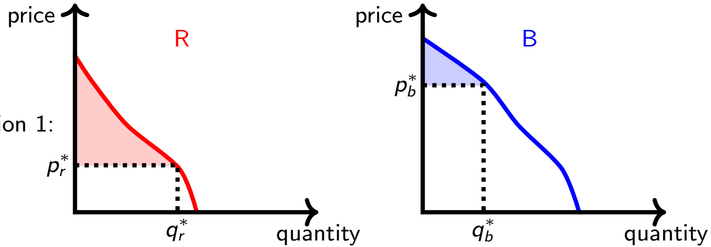

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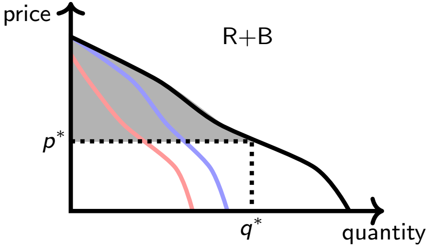

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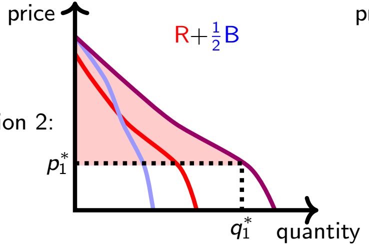

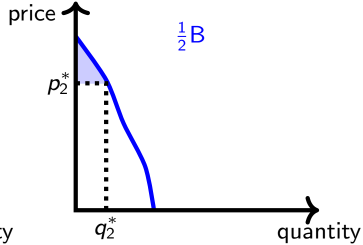

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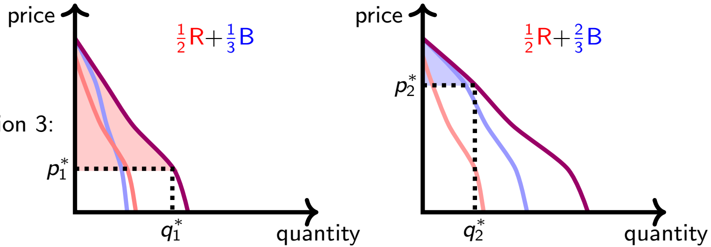
Segmentation 3:



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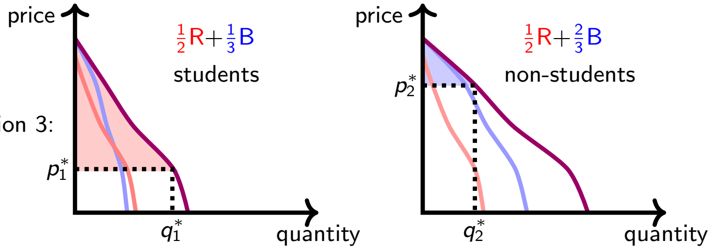

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When is "more price discrimination" bad?

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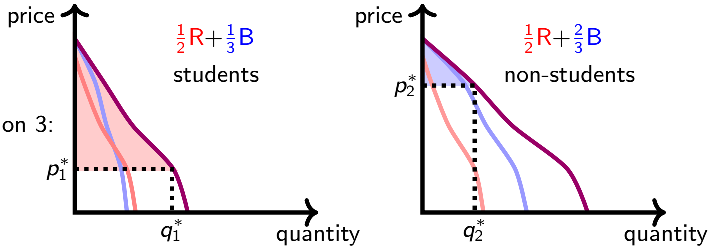

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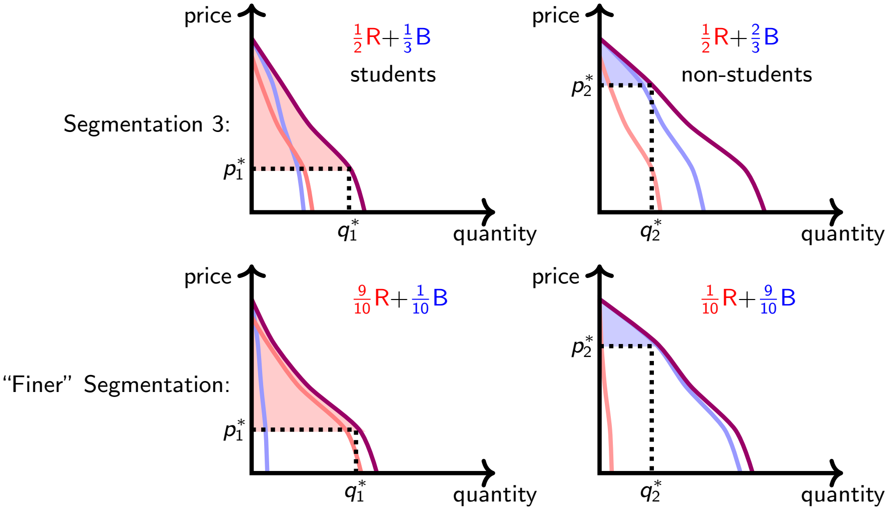

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When is “more price discrimination” bad?

When it is $\Rightarrow$ seller’s data usage must be banned
When it isn’t $\Rightarrow$ scope for appropriate regulation

Results

A characterization of when more price discrimination is bad.

- A separability condition
  - All demands must be decomposable into at most two demands
- Satisfied by linear demands
  - Linear demand $\Rightarrow$ more price discrimination is bad
  - Pigou's idea generalizes

Model

A family of demand curves $\mathcal{D} = \{D(p,\theta)\}_{\theta\in\Theta}$, a distribution $\mu \in \Delta(\Theta)$.

- Each downward sloping with concave revenue function.

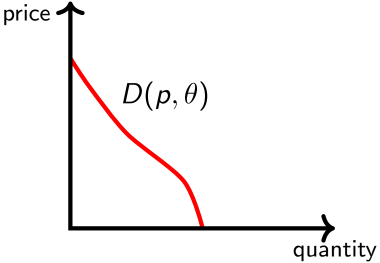
A segmentation: a distribution $f \in \Delta(\Delta(\Theta))$ over "markets" $\nu \in \Delta(\Theta)$.

- s.t. $E_f[\nu] = \mu$.
- Seller chooses a profit-maximizing price for every market $\nu \in f$.

"Information is always bad" (IAB) if for every two segmentations f, f'

if f is a garbling of f' (f' finer than f)

$\Rightarrow$ f gives a higher (α-weighted) surplus $V^\alpha = \alpha CS + (1-\alpha)R$.



Bergeman, Brooks, Morris 2015

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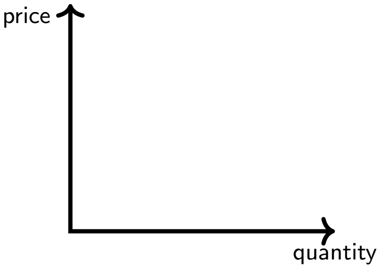

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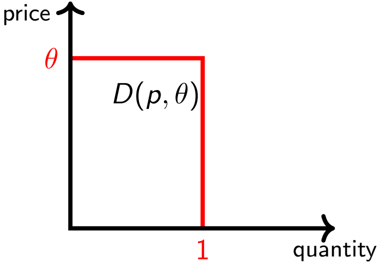

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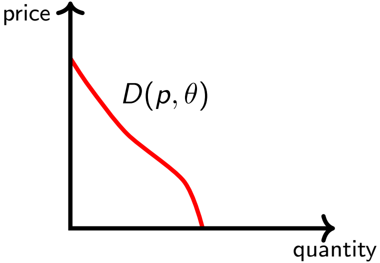

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Observation: If IAB holds for $\mathcal{D}$, then

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Observation: If IAB holds for $\mathcal{D}$, then

- $D(p^*(\theta'), \theta) > 0, \forall \theta, \theta'$. No-exclusion.

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     - (i) $D(p,\theta) = f_1(\theta)D_1(p) + f_2(\theta)D_2(p)$ *for all* θ *and*
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$$V_2(p) - V_1(p) + \frac{-\frac{R_1'(p)}{R_2'(p)}V_2' + V_1'}{-\frac{R_1'(p)}{R_2'(p)}R_2'' + R_1''}(R_1'(p) - R_2'(p))$$

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$V_i(p) = V_i^\alpha(p) = \alpha CS_i(p) + (1-\alpha)R_i(p).$

The second condition: interpretation

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Let μ_2 = measure of demand 2. Increase μ_2 slightly

$$\text{effect} = \Delta(p) := \underbrace{V_2(p) - V_1(p)}_{\text{composition effect}} + \underbrace{E[V_i'(p)]p'(\mu_2)}_{\text{price change effect}}$$

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If decreasing, then splitting μ_2 to $\frac{1}{2}(\mu_2 + \epsilon)$ and $\frac{1}{2}(\mu_2 - \epsilon)$ decreases value

The second condition: examples

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If f is log-concave $\Rightarrow \log p^2 f(p)$ is concave.
Uniform: $f(p) = 1$ is log-concave.
Edge case: $f(p) = a_1 \frac{a_2^p}{p^2}$ both IAB and IAG!

- D is a Gamma function.

Conclusions

A characterization of when no-segmentation is optimal.

- A strong separability condition
  - All demands must be decomposable into at most two demands
- Satisfied by linear demands
  - Pigou's intuition generalizes
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Thanks!

Related Literature

Segmented versus non-segmented, downward sloping demands:

- Pigou 1920; Robinson 1933; Varian 1985; Aguirre, Cowan, Vickers 2010

All segmentations, unit demands:

- Bergemann, Brooks, Morris 2014

Duality approaches in persuasion:

- Dworczak, Martini 2019; Kolotilin, Corrao, Wolitzky 2023; Smolin, Yamashita 2023; Dworczak, Kolotilin 2023