EXAM 1 REVIEW

Exam 1 will take place on Friday September 23rd during our scheduled lecture time (3:35-4:25pm in Doherty Hall 2210). Here is some information.

- The exam will cover material from Sections 2.1-2.4 and 3.1-3.3 of our text.
- You'll need a scientific calculator to perform some of the computations. Graphing calculators will not be permitted.
- You may bring notes on the front and back of a **half sheet of paper**.
- The exam will have three sections, including around 5-7 true or false questions, 3-5 short answer questions, and 4-6 free response questions.

This review will not be collected for credit. Solutions will be posted by Thursday before the exam. Note that the problems on this review are not comprehensive, make sure to also study the material from the course recommended below.

Tips for studying

I recommend the following strategy:

1. Start early
2. Understand every problem on this review
3. Review all worksheets (there are blank copies and solutions on my site)
4. Review relevant concept quizzes.
5. Review examples from lecture.
6. Review previous homework assignments.
7. Do some additional odd-numbered problems from our text (note that the answers are in the back of the book).

Topics

Here are some key words to help you study.

1. Chapter 2: Limits and Derivatives
    - The (informal) definition of a limit
    - One-sided limits
    - Limits that don't exist
    - Limit laws
    - Computing limits using algebra
    - Computing limits using squeeze theorem
    - The definition of continuity

- Types of discontinuities (removable, jump, and infinite)
- Intermediate value theorem (how to use this to show that an equation has a solution in a particular interval.

2. Chapter 3: Differentiation

   - Computing derivatives **by definition**
   - Finding equations of tangent lines
   - Sketching the graph of the derivative function
   - Functions that are not differentiable
   - Higher derivatives
   - Power, sum and constant multiple rules
   - Product and quotient rules

Practice Problems

True/False Questions:

1. True or False: The limit $\lim_{x \to 2} \dfrac{x^2 - 4}{x - 2}$ is undefined

2. True or False: A differentiable function must be continuous.

3. True or False: A continuous function is always differentiable.

4. True or False: If f and g are differentiable functions, then $(f/g)' = f'/g'$.

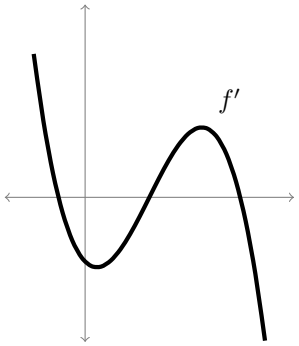
5. True of False: If the slope of the tangent line to the curve $y = f(x)$ at $x = 1$ is 3, then the slope of the tangent line to the curve $y = 2f(x)$ at $x = 1$ is 6.

6. True of False: If $f''(a) < 0$, then f is decreasing at $x = a$.

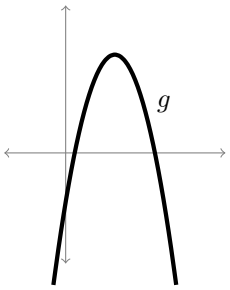
Short Answer Questions

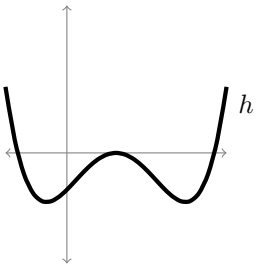
1. Let $f(x) = |x|$. Sketch the graph of f. Explain why f is not differentiable at $x = 0$.

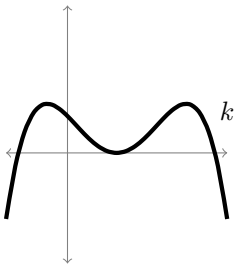
2. Let f be a differentiable function. The graph of its **derivative**, f' is given below



Consider the functions g, h and k graphed below. Identify which of these function could be the graph of f. Provide reasoning for your answer.







Free Response Questions:

1. Compute each of the following limits, or state that it does not exist.

   a) $\lim_{x \to 3^-} \dfrac{|x-3|}{x-3}$

   b) $\lim_{x \to \infty} \sin(x)$

   c) $\lim_{x \to 3} f(x)$, where $f(x) = \begin{cases} (x-3)^2 & \text{if } x < 3, \\ 9 - 3x & \text{if } x > 3. \end{cases}$

   d) $\lim_{x \to 0} \dfrac{1}{x^3}$

e) $\lim_{x \to 2} \dfrac{\sqrt{11 - x} - 3}{x - 2}$

f) $\lim_{x \to \infty} \dfrac{6}{e^x + 3}$

g) $\lim_{x \to e} \sqrt{\ln(x^3) + 1}$

h) $\lim_{x \to 0} \sin(1/x)$

i) $\lim_{x \to \infty} x^2 \sin(1/x)$

2. Use the definition of the derivative to find $f'(3)$ for $f(x) = \dfrac{2}{x+1}$.

3. Show that the polynomial $f(x) = x^4 - x^2 - 1$ has a root between $x = 1$ and $x = 2$.

4. Suppose that $f(1) = 3, g(1) = 2, f'(1) = 0, f'(2) = -1,\ f(2) = 0$, and $g'(1) = 5$. Compute $h'(1)$ for the following.

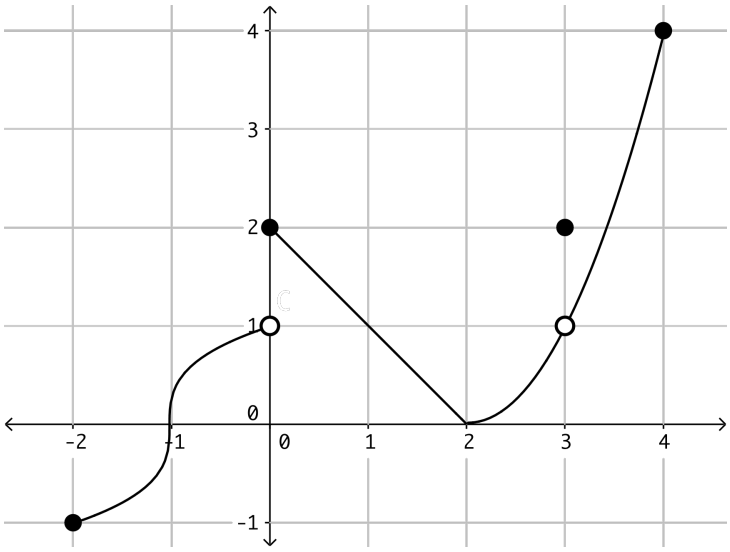
   a) $h(x) = \dfrac{f(x) - g(x)}{f(x)}$

   b) $h(x) = \sqrt{x}f(x) + g(x)$

5. Compute $f''(1)$ given $g(x) = 3x^2 - 2x^4 + 1$.

6. Find the equation of all horizontal tangent lines to $y = 2x^3 - 4x + 5$.

7. Let f be the graph given to the right.



a) Evaluate $\lim_{x \to 3} f(x)$

b) Evaluate $\lim_{x \to 0^+} f(x)$

c) Evaluate $f'(1)$

c) At what points is f not continuous? State the type of discontinuity (infinite, jump or removable).

d) At what points is f not differentiable? State why f is not differentiable there (corner or infinite tangent line).