EXAM 3 REVIEW

Exam 2 will take place on Monday, November 21st during our scheduled lecture time (3:35-4:25pm in Doherty Hall 2210). Here is some information.

- The exam will cover material from Sections 4.4 through 4.8 and 5.1 through 5.3 of our text.
- You'll need a scientific calculator to perform some of the computations. Graphing calculators will not be permitted.
- You may bring notes on the front and back of a **half sheet of paper**.
- The exam will have three sections, including around 5-7 true or false questions, 3-5 short answer questions, and 4-6 free response questions.

This review will not be collected for credit. Solutions will be posted by Friday before the exam. Note that the problems on this review are not comprehensive, make sure to also study the material from the course recommended below.

Tips for studying

I recommend the following strategy:

1. Start early.
2. Understand every problem on this review.
3. Review all worksheets (there are blank copies and solutions on my site).
4. Review relevant concept quizzes.
5. Review examples from lecture.
6. Review previous homework assignments.
7. Do some additional odd-numbered problems from our text (note that the answers are in the back of the book).

Topics

Here are some key words to help you study.

1. Chapter 4: Applications of Derivatives
    - The Mean Value Theorem and its consequences
    - Graph sketching
        - Finding local extrema with first and second derivative test
        - Concavity and inflection points
        - Horizontal and vertical asymptotes
    - L'Hopital's Rule
    - Optimization

2. Chapter 5: Integration
   - Approximating areas with Riemann sums
     - Left and right Riemann sums
   - Definition of the integral (as the area under a curve).
   - Computing the definite integral through area computations.
   - The Fundamental Theorem of Calculus I and II
     - Interpreting what the definite integral measures in the context of a real world problem
     - Computing the definite integral with antiderivatives

Practice Problems

True/False Questions:

1. True or False: If $f''(a) < 0$, then f is decreasing at $x = a$.

2. True or False: The function $f(x) = e^x + x - 1$ has two distinct roots.

3. True of False: If $f''(c) = 0$ then the function f changes concavity at $x = c$.

4. True of False: If f has a single critical point at $x = c$ and $f''(c) < 0$ then f has a global maximum at $x = c$.

5. True or False: If f is an increasing function on an interval $[a, b]$, then the left Riemann sum will always be an underestimate of the definite integral $\displaystyle\int_a^b f(x)dx$.

Short Answer Questions:

1. If a function has a single critical point on its domain, explain why it can have at most one root. What would need to be true about this function to guarantee it has exactly one root?

2. Explain what the following definite integrals represent in the given context.

   a) $\int_0^3 r(t)dt$, where $r(t)$ denote the rate at which water flows into a reservoir after t hours.

   b) $\int_3^5 f(t)dt$, where $f(t)$ measures the power consumption (that is, the rate of energy consumption) in Pittsburgh on a day in November t hours past midnight.

   c) $\int_1^3 a(t)dt$, where $a(t)$ denotes the acceleration of a car after t hours of driving.

Free Response

1. Compute the following limits.

   a) $\displaystyle\lim_{x\to 0} \frac{\log(x^2+1)}{x}$

   b) $\displaystyle\lim_{x\to 0} \frac{x-\sin(x)}{x-\tan(x)}$

   c) $\displaystyle\lim_{x\to\infty} (e^x + x)^{1/x}$

   d) $\displaystyle\lim_{x\to\infty} x\left(\frac{\pi}{2} - \tan^{-1}(x)\right)$

2. Find the vertical and horizontal asymptotes of

$$f(x) = \frac{e^{x+3} + e^x}{5e^x - 10e^{-x}}.$$

3. Speedometer readings for a motorcycle at 12-second intervals are given in the table.

t (s)	0	12	24	36	48	60
v (ft/s)	30	28	25	22	24	27

a) Estimate the distance traveled by the motorcycle during this time period using velocities at the beginning of the time intervals.

b) Give another estimate using the velocities at the end of the time periods.

4. A helicopter is rising straight up in the air. Its velocity at time t is $v(t) = 2t + 1$ feet per second. How high does the helicopter rise during the first 5 seconds?

5. Let $f(x) = 3x^4 + 4x^3$. Then we have

$$f'(x) = 12x^2(x+1) \text{ and } f''(x) = 12x(3x+2).$$

So, f has critical values at $x = 0$ and $x = 1$, and $f''(x) = 0$ when $x = 0$ and $x = -2/3$. Moreover, $f(0) = 0, f(1) = 7, f(-2/3) \approx -0.6$. Use this information to answer the following.

a) Find the intervals where f is increasing and decreasing.

b) Identify which critical values are local maximums and minimums.

c) Find the intervals where f is concave up and concave down.

d) Observe that $f(0) = 0$ and f has horizontal intercepts at $x = 0$ and $x = -4/3$ (you do not need to show this). Use this information, along with the previous parts, to sketch a graph of f.

6. A city on the north side of a 1/5-mile-wide river is 5 miles down river from another city on the south side of a river. Say that we want to build a highway to connect these two cities. If it costs \$8 million per mile to build on land and \$20 million per mile to build across land, describe the most economical way to build the highway.

7. A restaurant has a seating area that can seat up to 25 tables with 4 people per table. If they only put in 15 tables, they can charge an average \$20 per meal and fill the restaurant. However, for each additional table they put in they must decreasing the price of the average meal by \$1. Assuming all of the tables will be filled, what number of tables will maximize their profit?