

This Worksheet will be collected at the end of your recitation section on **Thursday, Sep 8th**.

2.4. Continuity

1. Find a value b such that the following function is continuous for all real numbers.

$$g(x) = \begin{cases} 2bx, & \text{if } x \leq -1, \\ 3x^2 + x + b, & \text{if } x > -1. \end{cases}$$

2. Compute the following limits

a) $\lim_{x \rightarrow -1} \frac{\ln(x^2)}{2x}$.

b) $\lim_{x \rightarrow \pi/4} (\sin(x) + \sqrt{\tan(x)})$

3. Determine which of the following functions are continuous. For those that are not, indicate what type of discontinuities they contain.

$$\text{a) } f(x) = \begin{cases} \cos(x^2 - 1) & \text{if } x < 1 \\ \sqrt{x} & \text{if } x \geq 1. \end{cases}$$

$$\text{b) } g(x) = \begin{cases} x^{-4} & \text{if } x < 1 \\ 2^x & \text{if } x \geq 1. \end{cases}$$

$$\text{c) } h(x) = \begin{cases} \frac{x^2 + 2x + 1}{x + 1} & \text{if } x \neq -1 \\ 5 & \text{if } x = -1. \end{cases}$$

4. Show that the function $f(x) = \ln(x + e^x) - 1$ has a zero.

5. Consider the function

$$f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0. \end{cases}$$

Observe that this function is continuous on the interval $(0, 1)$ and we have $f(0) = -1$ and $f(1) = 1$. So, $f(0) \leq 0 \leq f(1)$. The intermediate value theorem implies that there is a value a with $0 < a < 1$ such that $f(a) = 0$.

a) Sketch a graph of f .

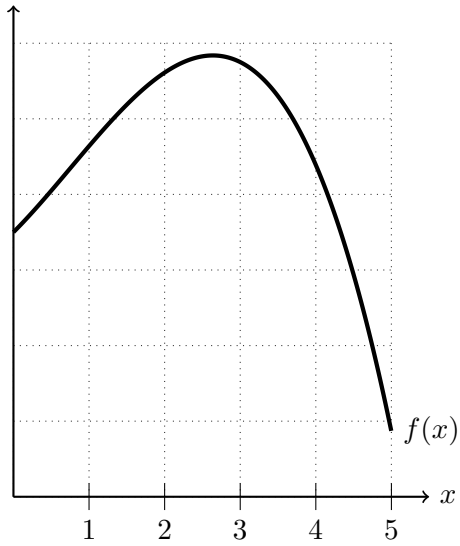
b) Observe from your graph that the conclusion we reached above is false! What went wrong?

3.1. Defining the Derivative

6. Let $f(x) = \frac{x+1}{x}$. Using only the definition, find the derivative $f'(1)$. Then, find the equation of the tangent line to f at the $x = 1$.

7. Let $g(x) = \sqrt{x+2}$. Using only the definition, find the derivative $g'(2)$. Then, find the equation of the tangent line to g at $x = 2$.

8. Use the graph of $f(x)$ to determine which is larger. Explain your answer.



a) $f(3)$ or $f(4)$?

b) $\frac{f(3) - f(1)}{3 - 1}$ or $\frac{f(3) - f(2)}{3 - 2}$?

c) $f'(1)$ or $f'(4)$?

d) $f'(5)$ or $f'(3)$?

Questionnaire:

Below are a few questions which are completely optional, and are meant to benefit you. Please only fill out what you feel comfortable with.

1. Is there anyone in class that you'd like to be grouped with next week?

2. Did you feel you worked well with your group this week?

3. Any other comments?

Grading Rubric:

Attendance: /20

Participation: /20

Completeness: /60