Name: ______________________________

Andrew ID: ________________________

General Instructions:

- This exam prep contains two sections: Section A includes computational problems, and Section B includes conceptual and proof-based problems. Please read the instructions at the beginning of each section carefully.
- You are allowed a full page of notes (front and back).
- No calculators or any other electronics are permitted.
- Write your answers clearly and make sure your handwriting is legible. If we cannot read your work, we will not grade the problem.
- All work must be completed in the space provided. If you need scratch paper, there is some at the front of the class. Please note that scratch work will not be graded.
- Please ask questions if anything is unclear.
- If you finish early, check all of your work, then bring this packet up to the front of the class.
- Good luck!

Section A.

INSTRUCTIONS:

1. Each problem in this section is worth **one point** and no partial credit will be given.
2. You do not need to show your work or provide justification on any problem in Section A.
3. Make sure to **clearly box or circle your answer**.

A1. This semester, you had to stop by the bookstore twice to purchase supplies for class. On the first day, you purchased two pens and a notebook for \$10. On the second day, you went back to the same bookstore and purchased the same pens and notebooks. This time, you purchased one pen and two notebooks, which came out to \$14. Assuming the price of the pens and notebooks remained the same between your two visits, how much does a single pen cost and how much does a single notebook cost?

Solution. Each pen costs \$2 and each notebook costs \$6.

A2. Find all solutions to the following system of linear equations

$$3x - 5y + z - \frac{1}{2}w = 1$$
$$5y + z - 2w = 0$$
$$2x + y + z + w = \frac{5}{3}$$
$$-10y - 2z + 4w = 3$$

Solution. This system has no solutions.

A3. Find all solutions to the system of linear equations with the following augmented matrix

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right)$$

Solution. The solution set is given by

$$(x, y, z, w) = (5, t, 2 - t, 1)$$

for all real numbers t.

Section B.

Instructions:

1. Each problem in this section is worth **5 points**.
2. You must provide justification for all of your answers in Section B.
3. Points will be awarded based on the rubric below. Note that half points may be awarded, and further rubric items may be added to cover potential cases not outlined below.

Points	**Rubric**
5	Solution is presented with clear justification that is logically complete and correct. May include minor typos and computational errors if they do not majorly impact the argument. No important steps are missing or assumed. All assumptions and special cases have been covered. All suggestions for improvement come under the category of "improvements for clarity" rather than "correcting logical errors". Omission of details will be judged depending on context of the material, with simpler steps being acceptable for omission when covering more advanced topics.
4	Solution is close to full and complete, but contains either a computational error or an error in reasoning that majorly impacts the argument. This score is also appropriate for solutions that are mathematically sound but confusingly written.
3	Solution is incorrect, but understanding of the problem was demonstrated and student provided a clear outline of a potential approach with information about where they got stuck **-or-** solution is correct but justification is insufficient or so confusingly written that it cannot be followed with a reasonable amount of effort.
2	Solution is incorrect, but student demonstrated understanding of the problem **-or-** solution is correct and student did not provide justification for their answer.
1	Solution is incorrect and student did not demonstrate understanding of the problem, but did demonstrate some knowledge of relevant material.
0	Solution is incorrect or incomplete, and there was no demonstration of knowledge of relevant material.

B1. Recall that a linear equation in three variables defines a plane in $\mathbb{R}^3$. Is it possible for two linear equations in three unknowns to have exactly one solution? Justify your answer **using only geometric reasoning**.

Solution. No, this is not possible. Two linear equations in three unknowns corresponds to the intersection points of two planes in in $\mathbb{R}^3$. If our two planes are equal then the system has infinitely many solutions. If the two planes are distinct and parallel then the system has no solutions. If the two planes are distinct and not parallel, they intersect at a line, and so the system has infinitely many solutions.

B2. Determine whether the following statement is true or false: if there is a row of zeros in the augmented matrix for a linear system, then it must have infinitely many solutions. **If true, provide a proof. If false, provide a counterexample and justify why this is a counterexample.**

Solution. This is false, since our system could have no solutions. For example, the augmented matrix

$$\left(\begin{array}{cc|c} 0 & 0 & 1 \\ 0 & 0 & 0 \end{array}\right)$$

corresponds to a system with no solutions.

B3. Recall that a system of linear equations is called homogeneous if the constant term in each equation is equal to zero. For example,

$$2x + y = 0$$
$$3x - y = 0$$

is a homogeneous system of linear equations. Determine whether the following statement is true or false: every homogeneous system has exactly one solution. **If true, provide a proof. If false, provide a counterexample and justify why this is a counterexample.**

Solution. This is false, since our system could have infinitely many solutions. For example, the augmented matrix

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

corresponds to a homogeneous system with infinite solution set

$$(x, y, z) = (t, -t, 0).$$