EXAM PREP PREP SOLUTIONS

Name: ______________________________

Andrew ID: ________________________

GENERAL INSTRUCTIONS:

- This exam prep contains two sections: Section A includes computational problems, and Section B includes conceptual and proof-based problems. Please read the instructions at the beginning of each section carefully.
- You are allowed a full page of notes (front and back).
- No calculators or any other electronics are permitted.
- Write your answers clearly and make sure your handwriting is legible. If we cannot read your work, we will not grade the problem.
- All work must be completed in the space provided. If you need scratch paper, there is some at the front of the class. Please note that scratch work will not be graded.
- Please ask questions if anything is unclear.
- If you finish early, check all of your work, then bring this packet up to the front of the class.
- Good luck!

Section A.

INSTRUCTIONS:

1. Each problem in this section is worth **one point** and no partial credit will be given.
2. You do not need to show your work or provide justification on any problem in Section A.
3. Make sure to **clearly box or circle your answer**.

A.1. Convert the following situation into a system of linear equations. **Do not solve the system!**. A furniture company makes chairs, coffee tables, and desks. Each chair requires 10 minutes of sanding, 6 minutes of staining, and 12 minutes of varnishing. Each table requires 12 minutes of sanding, 8 minutes of staining, and 12 minutes of varnishing. Finally, each desk requires 15 minutes of sanding, 12 minutes of staining, and 18 minutes of varnishing. Each week, the sanding bench is available for 16 hours, the staining bench for 11 hours, and the varnishing bench for 18 hours. How many of each piece of furniture should be made each week so that the benches are fully utilized? **You do not need to provide a justification for your answer, but make sure to clearly label your variables.**

Solution. Let x denote the number of chairs, y the number of coffee tables, and z the number of desks. To fully utilize the benches, we should find the value(s) x, y and z that satisfy the following system of linear equations

$$10x + 12y + 15z = 60 \cdot 16$$

$$6x + 8y + 12z = 60 \cdot 11$$

$$12x + 12y + 18z = 60 \cdot 18$$

A2. Find all solutions to the following system of linear equations

$$x + y + z = 2$$

$$x - y + z = 1$$

$$x - y - z = 0$$

Solution. This system has one solution given by

$$(x, y, z) = \left(1, \frac{1}{2}, \frac{1}{2}\right)$$

A3. Find all solutions to the system of linear equations with the following augmented matrix

$$\left(\begin{array}{cccc|c} 2 & 3 & -1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 2 \\ -1 & 0 & -1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 4 \end{array}\right)$$

Solution. This system has no solutions.

Section B.

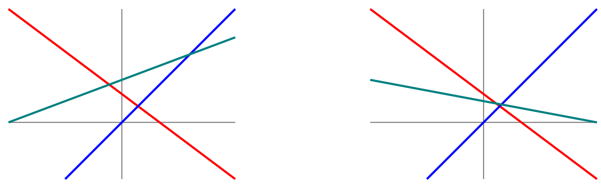
INSTRUCTIONS:

1. Each problem in this section is worth **5 points**.
2. You must provide justification for all of your answers in Section B.
3. Points will be awarded based on the rubric below. Note that half points may be awarded, and further rubric items may be added to cover potential cases not outlined below.

Points	**Rubric**
5	Solution is presented with clear justification that is logically complete and correct. May include minor typos and computational errors if they do not majorly impact the argument. No important steps are missing or assumed. All assumptions and special cases have been covered. All suggestions for improvement come under the category of "improvements for clarity" rather than "correcting logical errors". Omission of details will be judged depending on context of the material, with simpler steps being acceptable for omission when covering more advanced topics.
4	Solution is close to full and complete, but contains either a computational error or an error in reasoning that majorly impacts the argument. This score is also appropriate for solutions that are mathematically sound but confusingly written.
3	Solution is incorrect, but understanding of the problem was demonstrated and student provided a clear outline of a potential approach with information about where they got stuck **-or-** solution is correct but justification is insufficient or so confusingly written that it cannot be followed with a reasonable amount of effort.
2	Solution is incorrect, but student demonstrated understanding of the problem **-or-** solution is correct and student did not provide justification for their answer.
1	Solution is incorrect and student did not demonstrate understanding of the problem, but did demonstrate some knowledge of relevant material.
0	Solution is incorrect or incomplete, and there was no demonstration of knowledge of relevant material.

B.1. Recall that a linear equation in two variables defines a line in $\mathbb{R}^2$. Suppose that you have a system of three linear equations in two variables where all three of the corresponding lines are distinct. **Using only geometric reasoning**, explain why this system cannot have infinitely many solutions.

Solution. Recall that a linear equation in two variables defines a line in $\mathbb{R}^2$. Since we've assumed all three of our lines are distinct we have two cases: none of our lines are parallel, or two or more of our lines are parallel. If none of our three lines are parallel, then there are either no intersection points, or one intersection point, as pictured below



Since two parallel lines do not intersect, then there are no intersection points when two or more of our lines are parallel. So, in all cases our system either has exactly one solution or no solutions.

B.2. Determine whether the following statement is true or false: if a linear system has fewer equations than variables, it must have an infinite number of solutions. **If true, provide a proof. If false, provide a counterexample and justify why this is a counterexample.**

Solution. This is $\boxed{\text{False}}$, since we could have a linear system with no solutions. For example, we could have the linear system

$$x + y + z = 1$$

$$x + y + z = 3$$

If we replace the second equation with the second equation minus the first equation we see this system is equivalent to

$$x + y + z = 1$$

$$0 = 2$$

and so this system has no solutions. (Note that the statement is true if our system is known to have at least one solution).

B.3. A system of linear equations is called HOMOGENEOUS if the constant term in each equation is zero. For example

$$2x + y = 0$$

$$3x - y = 0$$

is a homogeneous system of linear equations. Is it possible for a homogeneous system of linear equations to have no solutions? If so, provide an example. If not, provide justification for why not.

Solution. No, this is not possible. That is, *every* homogeneous system of linear equations has at least one solution, which holds because every homogeneous system has the solution with all variables equal to 0.