$\boxed{P1.}$

$$\begin{cases} x + 2y - 3z = 0 \\ 2x - y + z = 0 \\ 3x - y + z = 4 \end{cases}$$

Replace (eq. 2) by
(eq. 2) − 2 (eq. 1):

$$\begin{cases} x + 2y - 3z = 0 \\ -5y + 7z = 0 \\ 3x - y + z = 4 \end{cases}$$

Replace (eq. 3) by
(eq. 3) − 3 (eq. 1):

$$\begin{cases} x + 2y - 3z = 0 \\ \quad -5y + 7z = 0 \\ \quad -7y + 10z = 4 \end{cases}$$

Replace (eq. 3) by

(eq. 3) $- \frac{7}{5}$ (eq. 2):

$$\begin{cases} x + 2y - 3z = 0 \\ \quad -5y + 7z = 0 \\ \quad \quad \frac{z}{5} = 4 \end{cases}$$

$$\Rightarrow \quad z = 20$$

Now substitute $z = 20$ into eqs. 1 and 2:

$$\begin{cases} x + 2y = 60 \\ -5y = -140 \end{cases}$$

$$\Rightarrow y = \frac{-140}{-5} = 28$$

Substituting into eq. 1:

$$x = 60 - 2 \cdot 28 = 4$$

So, the unique solution is

$x = 4$, $y = 28$, $z = 20$

P2.

$$\begin{cases} x + 2y = -1 \\ 2x + y + z = 1 \\ -x + y - z = -1 \end{cases}$$

Replace (eq 2) by (eq 2) − 2(eq 1):

$$\begin{cases} x + 2y = -1 \\ -3y + z = 3 \\ -x + y - z = -1 \end{cases}$$

Replace (eq 3) by (eq 3) + (eq 1):

$$\begin{cases} x + 2y = -1 \\ -3y + z = 3 \\ 3y - z = -2 \end{cases}$$

Replace (eq 3) by (eq 3) + (eq 2):

$$\begin{cases} x + 2y = -1 \\ -y + z = 3 \\ 0 = 1 \end{cases}$$

The last equation, $0 = 1$, is absord, so the system has no solutions.

P3.

$$\begin{cases} 2x + y = 3 \\ 4x + y = 7 \\ 2x + 5y = -1 \end{cases}$$

Remark: This system has more equations than variables.

Replace (eq 2) by (eq 2) − 2(eq 1):

$$\begin{cases} 2x + y = 3 \\ \quad\; -y = 1 \\ 2x + 5y = -1 \end{cases}$$

From eq. 2, $y = -1$.

Therefore,

$$\begin{cases} 2x - 1 = 3 \\ 2x - 5 = -1 \end{cases}$$

Both equations now become: $2x = 4$,

so $x = 2$.

The unique solution is

$x = 2$, $y = -1$.

P4.

$$\begin{cases} w + x + 2y + z = 1 \\ w - x - y + z = 0 \\ x + y = -1 \\ w + x + z = 2 \end{cases}$$

Replace (eq 2) by (eq 2) − (eq 1):

$$\begin{cases} w + x + 2y + z = 1 \\ -2x - 3y = -1 \\ x + y = -1 \\ w + x + z = 2 \end{cases}$$

Interchange eq 2 and eq 3:

$$\begin{cases} w + x + 2y + z = 1 \\ x + y = -1 \\ -2x - 3y = -1 \\ w + x + z = 2 \end{cases}$$

Replace (eq. 3) with

(eq. 3) + 2 (eq. 2):

$$\begin{cases} w + x + 2y + z = 1 \\ x + y = -1 \\ -y = -1 \\ w + x + z = 2 \end{cases}$$

$\Rightarrow y = 1$, which we can now substitute into the other equations:

$$\begin{cases} w + x + 2 + z = 1 \\ x + 1 = -1 \\ w + x + z = 2 \end{cases}$$

$\Rightarrow x = -2$ from (eq 2).

Replacing $x = -2$ into the other two eqns:

$$\begin{cases} w - 2 + 2 + z = 1 \\ w - 2 + z = 2 \end{cases}$$

$$\Rightarrow \begin{cases} w + z = 1 \\ w + z = 4 \end{cases}$$

Both equations can't be true at the same time (otherwise $1 = 4$), hence the system has no solutions.