WORKSHEET 3 NAME:

This Worksheet will collected at the end of your recitation section and will be graded on completeness. We will return your graded worksheet back to you during recitation next week.

Chapter 3. Vector Spaces

P1. Let $V = \text{Span}(\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4)$ be a vector subspace of $\mathbb{R}^n$, and let

$$A = \begin{pmatrix} \vec{v}_1 & \vec{v}_2 & \vec{v}_3 & \vec{v}_4 \end{pmatrix}.$$

Suppose that the reduced row echelon form of A has a pivot in columns 1, 3 and 4, and no pivot in column 2.

a) Show that $\vec{v}_2$ is in $\text{Span}(\vec{v}_1, \vec{v}_3, \vec{v}_4)$ *(Hint: think about which variable is free in the vector equation $x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3 + x_4\vec{v}_4 = \vec{0}$).*

b) Explain how we know that $\{\vec{v}_1, \vec{v}_3, \vec{v}_4\}$ is a linearly independent set.

c) Conclude that $\{\vec{v}_1, \vec{v}_3, \vec{v}_4\}$ is a basis for V.

P2. Find a basis and the dimension of the following vector subspaces of $\mathbb{R}^3$.

a) $V = \operatorname{Span}\left(\begin{pmatrix}1\\1\\0\end{pmatrix}\begin{pmatrix}2\\1\\1\end{pmatrix}, \begin{pmatrix}1\\2\\-1\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}\right).$

b) $W = \operatorname{Span}\left(\begin{pmatrix}1\\-2\\2\end{pmatrix}, \begin{pmatrix}2\\-3\\4\end{pmatrix}, \begin{pmatrix}0\\4\\0\end{pmatrix}, \begin{pmatrix}-1\\5\\-2\end{pmatrix}\right).$

P3. Let $V = \text{Span}(\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m)$ be a vector subspace of $\mathbb{R}^n$ and let

$$A = \begin{pmatrix} \vec{v}_1 & \vec{v}_2 & \cdots & \vec{v}_m \end{pmatrix}.$$

Suppose that the reduced row echelon form has a pivot in k columns. What is the dimension of V? Explain how you know this.

P4. Let $A = \begin{pmatrix} \vec{v}_1 \cdots \vec{v}_m \end{pmatrix}$ where $\vec{v}_i$ are vectors in $\mathbb{R}^n$. Suppose that the reduced row echelon form of A has a pivot in every **row**. Show that we have $\text{Span}(\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m) = \mathbb{R}^n$.

P5. Determine which of the following sets are bases for $\mathbb{R}^4$.

a) $\mathcal{A} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix} \right\}$

b) $\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix} \right\}.$

c) $\mathcal{C} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$, where the $\vec{v}_i$ are any vectors in $\mathbb{R}^4$

d) $\mathcal{D} = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4, \vec{v}_5\}$, where the $\vec{v}_i$ are any vectors in $\mathbb{R}^4$.

Questionnaire:

Below are a few questions which are completely optional, and are meant to benefit you. Please only fill out what you feel comfortable with.

1. What did you think of the worksheet this week (length, difficulty, etc)?

2. Did you feel you worked well with your group this week?

3. Any other comments?

Grading Rubric:

Participation:	/40
Completeness:	/60