Instructions: Complete all problems from the list below. This assignment will be due on Gradescope no later than **7pm on Tuesday, September 6th**. Late work will not be accepted. There will be no exceptions for technology issues, so I suggest you upload your homework at least one hour before the deadline. Please make sure you've done all of the following before submitting your work:

* **Do not** write your name anywhere on your submission. Gradescope will keep track of your submission, and will allow me to use a blind grading process.
* Type your homework using LaTeX.
* Write up proofs formally and completely.
* If you use any resources (stackexchange, tutors, friends), please include a list of references in your writeup.

Problems: 1-8 from Chapter 1 of the Lecture Notes.

1. Prove Lemma 1.4 from the Lecture Notes.
2. Prove Lemma 1.5 from the Lecture Notes. (Hint: use strong induction).
3. Prove Lemma 1.6 from the Lecture Notes.
4. Complete the proof of Lemma 1.7 from the Lecture Notes. That is, show that

$$\mathrm{lcm}(a, b) = \prod_{i=1}^{\infty} p_i^{\max\{k_i, r_i\}}.$$

   (Hint: observe that we can write $\max\{k_i, r_i\} = k_i + k_i'$ and $\max\{k_i, r_i\} = r_i + r_i'$ where $k_i', r_i' \geq 0$.)

5. For each pair of integers, compute $\gcd(a, b)$ and $\mathrm{lcm}(a, b)$ using any method discussed in Chapter 1. Make sure to show your work.
   a) $a = 256, b = 160$
   b) $a = 7544, b = 115$
   c) $a = 8633, b = 8051$
6. This problem will complete the proof of Lemma 1.8 from the Lecture Notes. For integers x, y, show that $\min\{x, y\} + \max\{x, y\} = x + y$.
7. Let a, b, c be positive integers. If $a \mid bc$ and $\gcd(a, b) = 1$, show that $a \mid c$.
8. Prove that there are infinitely prime numbers p with $p \equiv 1 \pmod{4}$.