Instructions: Complete all problems from the list below. This assignment will be due on Gradescope no later than **7pm on Friday, December 9th**. Late work will not be accepted. There will be no exceptions for technology issues, so I suggest you upload your homework at least one hour before the deadline. Please make sure you've done all of the following before submitting your work:

* **Do not** write your name anywhere on your submission. Gradescope will keep track of your submission, and will allow me to use a blind grading process.
* Type your homework using LaTeX.
* Write up proofs formally and completely.
* If you use any resources (stackexchange, tutors, friends), please include a list of references in your writeup.

Chapter 5 Problems:

15. Show that for two principal ideals $(x), (y)$ in an integral domain R, if $(x) = (y)$ then $x = uy$ for a unit u.
16. Show that an element p in an integral domain R is prime if and only if the ideal (p) is prime.
17. Use Minkowski's Convex Body Theorem to show that every prime $p \equiv 1 (\bmod 4)$ is a sum of two integer squares by considering a lattice with elements of the form (x_1, x_2) where $x_2 \equiv ux_1 (\bmod p)$ and u is any integer with $u^2 \equiv 1 (\bmod p)$.
18. This problem will help finish the proof of Theorem 5.67. Show that if $\mathfrak{a}$ and $\mathfrak{b}$ are ideals in $\mathcal{O}_K$ with $\mathfrak{a} \subseteq \mathfrak{b}$, then there exists an ideal $\mathfrak{c}$ so that $\mathfrak{c}\mathfrak{b} = \mathfrak{a}$. For this reason, we sometimes use the notation $\mathfrak{b} \mid \mathfrak{a}$ to mean $\mathfrak{a} \subseteq \mathfrak{b}$.
19. Prove Theorem 5.68; that is, show there are only finitely many ideals with a fixed norm. Conclude that the class group of any number field is finite. *(Hint: use unique factorization in $\mathcal{O}_K$ and Theorem 5.57).*