Instructions: Complete all problems from the list below. This assignment will be due on Gradescope no later than **7pm on Monday, September 12th**. Late work will not be accepted. There will be no exceptions for technology issues, so I suggest you upload your homework at least one hour before the deadline. Please make sure you've done all of the following before submitting your work:

* **Do not** write your name anywhere on your submission. Gradescope will keep track of your submission, and will allow me to use a blind grading process.
* Type your homework using LaTeX.
* Write up proofs formally and completely.
* If you use any resources (stackexchange, tutors, friends), please include a list of references in your writeup.

Problems: Solve the following problems from Chapter 2 of the Lecture Notes.

1. Prove Theorem 2.3.
2. Show that μ is multiplicative, but not completely multiplicative.
4. Determine if the following statement is true: if f is multiplicative, then

$$F(n) = \prod_{d|n} f(d)$$

   is also multiplicative. Provide a proof or counterexample.
6. Prove Corollary 2.9.
8. Determine which of the following statements are true. Provide a proof or counterexample.
   a) If $\gcd(m, n) = 1$ then $\gcd(\varphi(m), \varphi(n)) = 1$.
   b) If n is composite, then $\gcd(n, \varphi(n)) > 1$.
   c) If the set of distinct primes dividing m and the set of distinct primes dividing n are equal, then $n\varphi(m) = m\varphi(n)$.
10. For a fixed positive integer k, show that if the equation $\varphi(n) = k$ has only one integer solution $n > 0$, then $36 \mid n$.
11. For a fixed positive integer k, show that the equation $\varphi(n) = k$ has only finitely many integer solutions $n > 0$.
13. Prove Theorem 2.11 (you may assume the set of arithmetic functions forms a group under addition). That is, for arithmetic functions f and g, show that
   a) $f * g = g * f$,
   b) $(f * g) * h = f * (g * h)$,
   c) $f * I = f$, and
   d) $f * (g + h) = f * g + f * h$.