Instructions: Complete all problems from the list below. This assignment will be due on Gradescope no later than **7pm on Monday, September 19th**. Late work will not be accepted. There will be no exceptions for technology issues, so I suggest you upload your homework at least one hour before the deadline. Please make sure you've done all of the following before submitting your work:

* **Do not** write your name anywhere on your submission. Gradescope will keep track of your submission, and will allow me to use a blind grading process.
* Type your homework using LaTeX.
* Write up proofs formally and completely.
* If you use any resources (stackexchange, tutors, friends), please include a list of references in your writeup.

Problems: Solve the following problems from Chapter 2 of the Lecture Notes.

3. Recall that $\lfloor x \rfloor$ denotes the largest integer m with $m \leq x$. Let
$$f(n) = \lfloor \sqrt{n} \rfloor - \lfloor \sqrt{n-1} \rfloor.$$
Show that f is multiplicative but not completely multiplicative.

14. This problem will complete the proof of Lemma 2.16. Show that
$$(p^d - 1) \mid (p^n - 1)$$
if and only if $d \mid n$.

15. Compute $\Phi_{12}(x)$ and use this to find an expression for the four 12th roots of unity in terms of radicals.

16. Suppose that n and k are positive integers with prime factorizations
$$n = p_1^{a_1} \cdots p_t^{a_t} \text{ and } k = p_1^{b_1} \cdots p_t^{b_t}$$
with $a_i, b_i \geq 1$. Show that $\Phi_n(x^k) = \Phi_{nk}(x)$.

17. Prove the converse of Theorem 2.20. That is, show that for a multiplicative function f, if
$$f^{-1}(n) = \mu(n) f(n)$$
for all $n \geq 1$ then f is completely multiplicative. (Hint: use part (2) of Theorem 2.3).

18. For a multiplicative function f, prove the following:
    a) $f^{-1}(n) = \mu(n) f(n)$ for every square-free integer $n \geq 1$;
    b) $f^{-1}(p^2) = f(p)^2 - f(p^2)$ for every prime p.

19. Suppose that f is multiplicative. Prove that f is completely multiplicative if and only if $f^{-1}(p^a) = 0$ for all primes p and integer $n \geq 2$.