Instructions: Complete all problems from the list below. This assignment will be due on Gradescope no later than **7pm on Monday, September 26th**. Late work will not be accepted. There will be no exceptions for technology issues, so I suggest you upload your homework at least one hour before the deadline. Please make sure you've done all of the following before submitting your work:

* **Do not** write your name anywhere on your submission. Gradescope will keep track of your submission, and will allow me to use a blind grading process.
* Type your homework using LaTeX.
* Write up proofs formally and completely.
* If you use any resources (stackexchange, tutors, friends), please include a list of references in your writeup.

Chapter 2 Problems:

20. Give an elementary proof that the divisor functions $\sigma_k(n)$ are multiplicative. Show that $\sigma_k(n)$ is not completely multiplicative.

22. Prove Lemma 2.29 from the Lecture notes. That is, for a prime p, show that

$$\mu_p(x) = 1 - x, \mu_p^2(x) = 1 + x, \text{ and } \mathbf{1}_p(x) = \frac{1}{1-x}.$$

Chapter 3 Problems:

1. Show that

$$\prod_{k=0}^{n-1} F_k = F_n - 2,$$

for any integer $n \geq 1$. Use this to conclude that there are infinitely many prime numbers.

Bonus Problem: Given a nonconstant polynomial $f \in \mathbb{Z}[x]$, Schur's theorem tells us that the sequence $\{f(n)\}$ has infinitely many prime divisors. I conjecture that (maybe in all cases, but at least in some large family of cases), the Zsigmondy set $Z(\{f(n)\})$ is infinite. What ideas do you have to prove or disprove this conjecture?