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**Instructions:** Complete all problems from the list below. This assignment will be due on Gradescope no later than **7pm on Wednesday, November 2nd**. Late work will not be accepted. There will be no exceptions for technology issues, so I suggest you upload your homework at least one hour before the deadline. Please make sure you've done all of the following before submitting your work:

- \* **Do not** write your name anywhere on your submission. Gradescope will keep track of your submission, and will allow me to use a blind grading process.
- \* Type your homework using LaTeX.
- \* Write up proofs formally and completely.
- \* If you use any resources (stackexchange, tutors, friends), please include a list of references in your writeup.

**Chapter 5 Problems:**

1. Show that the minimal polynomial  $m_\alpha(X)$  of an algebraic number  $\alpha$  is unique. (*Hint: use the division algorithm in  $\mathbb{Q}[X]$* )
3. Show that the following complex numbers are algebraic. Determine which are algebraic integers. (Bonus: also find their minimal polynomials).
  - a)  $(1 + i)/\sqrt{2}$
  - b)  $i + \sqrt{2}$
  - c)  $e^{2\pi i/3} + 2$
  - d)  $\sqrt{1 + \sqrt{2}} + \sqrt{1 - \sqrt{2}}$
4. Show that the set of algebraic numbers  $\bar{\mathbb{Z}}$  is countable. Conclude that there exists infinitely many transcendental numbers.
5. Let  $K = \mathbb{Q}(\sqrt{2}, \sqrt[3]{2})$ . Find all monomorphisms  $K \hookrightarrow \mathbb{C}$  that fix  $\mathbb{Q}$ .