Instructions: Complete all problems from the list below. This assignment will be due on Gradescope no later than **7pm on Wednesday, November 30th**. Late work will not be accepted. There will be no exceptions for technology issues, so I suggest you upload your homework at least one hour before the deadline. Please make sure you've done all of the following before submitting your work:

* **Do not** write your name anywhere on your submission. Gradescope will keep track of your submission, and will allow me to use a blind grading process.
* Type your homework using LaTeX.
* Write up proofs formally and completely.
* If you use any resources (stackexchange, tutors, friends), please include a list of references in your writeup.

Chapter 5 Problems:

9. Let $K = \mathbb{Q}(\sqrt{D})$ for a square-free integer D. Show that
$$\mathcal{O}_K = \begin{cases} \mathbb{Z}[\sqrt{D}], & \text{if } D \not\equiv 1 (\operatorname{mod} 4) \\ \mathbb{Z}\left[\frac{1+\sqrt{D}}{2}\right], & \text{if } D \equiv 1 (\operatorname{mod} 4). \end{cases}$$
(Hint: observe that in the quadratic case, α is an algebraic integer if and only if $N_K(\alpha)$ and $Tr_K(\alpha)$ are both integers).
10. Show that $2 \cdot 3 = (1+\sqrt{-5})(1-\sqrt{-5})$ gives two distinct factorizations of 6 into irreducibles in the ring $\mathbb{Z}[\sqrt{-5}]$.
11. Show that the ideal (X) is prime but not maximal in $\mathbb{Z}[X]$.
12. This problem will finish the proof of Lemma 5.49.
    a) For a commutative ring R, show that an ideal $\mathfrak{a} \subseteq R$ is prime if and only if $R/\mathfrak{a}$ is an integral domain.
    b) Show that any finite integral domain is a field.
    c) Conclude that every prime ideal in $\mathcal{O}_K$ is maximal.
13. Let $R = \mathbb{Z}[\sqrt{-5}]$ and consider the ideals
$$\mathfrak{p} = (2, 1+\sqrt{-5})$$
$$\mathfrak{q} = (3, 1+\sqrt{-5})$$
$$\mathfrak{r} = (3, 1-\sqrt{-5}).$$
Show that these ideals are maximal (and hence prime). Furthermore, show that
$$\mathfrak{p}^2 = (2), \mathfrak{q}\mathfrak{r} = (3)$$
$$\mathfrak{p}\mathfrak{q} = (1+\sqrt{-5}), \mathfrak{p}\mathfrak{r} = (1-\sqrt{-5}).$$
14. Using the previous problem, show that the distinct factorizations into irreducibles from Problem 10 comes from two different groupings of the factorization into prime ideals $(6) = \mathfrak{p}^2\mathfrak{q}\mathfrak{r}$.